James Clayton Lecture

GYROSCOPES FOR INERTIAL NAVIGATION

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INTRODUCTION

THE PRACTICAL APPLICATION of the inertial properties of matter to defining the horizon by means of a gyroscope probably dates back to the eighteenth century (1)†, and the beginning of this century saw the first direct application to navigation, in the form of the gyrocompass. The navigation of a vehicle solely from information derived by inertial means is, however, of much more recent origin, dating from the German rocket developments that led to the V2 (2).

Since that date there have been rapid advances in technology that have led to widespread application in the missile, military aircraft, and naval fields. This advance has involved expenditure of very large sums of money both in development and on highly specialized capital facilities, and measured in capital cost per pound weight, inertial navigation equipment must rank as one of the most expensive articles manufactured today.

Where consistent operational performance is required over a long period of time, maintenance costs can also be heavy, and a wider use of inertial navigation, for example in civil aviation, is likely to be dependent on a reduction in the total cost to the user (3) (4) (5).

The basic reason for the high cost is that measurement is being made, to a high order of accuracy in a difficult environment, of very small forces and displacements. This poses most difficult problems both in the mechanical and electrical design of the measuring instruments themselves, and in the instrumentation necessary for translating the output into navigational information.

Basically two measuring instruments are required: an accelerometer to measure the combined force of gravity and acceleration, and an inertial directional reference (gyroscope); especially for non-missile applications the performance—and cost—is dominated by the gyroscope (4).

The objective of this lecture is to compare, on a common basis, the many possible ways of designing a gyroscope for inertial navigation, and to demonstrate the type of mechanical engineering problems that are encountered with present-day instruments in obtaining consistent performance over long periods at an acceptable cost.

Part I: The nature of the gyroscope problem

THE BASIC PRINCIPLES OF INERTIAL NAVIGATION

Newton's laws of motion state that, if a body is in a state of uniform motion in a straight line, it will continue in that state of motion unless acted upon by a force. Inertial navigation, together with the whole of our concept of dynamics, depends on this fundamental principle.

If the motion of a body is uniform and linear when referred (Fig. 1) to a set of axes OX_IY_I‡, it will not, however, be uniform and linear when referred to another set of axes OXY rotating relative to the first; it is therefore necessary to define the state of rotation of the reference axes before applying Newton's laws.

In choosing a set of reference axes any origin may be selected, provided that the gravitational field appropriate to the choice of origin is specified. If, for example, the origin is located at the mass centre of a satellite orbiting free of propulsion or drag, then the gravitational field at the origin is zero.

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† *References are given in the Appendix.*

‡ *Two-dimensional motion is illustrated for clarity; the extension to three dimensions follows the same basic principle.*

In defining direction of the axes, for all practical purposes, a set of axes fixed in direction relative to the stars can be used; if the origin is then selected so that the gravitational field there is zero then the axes are termed 'inertial'. Provided that the gravitational field is known at all other points, the change in motion of a body in inertial axes can then be determined by two types of measurement with inertial sensors.

Measurement of axis rotation—the gyroscope

If a mass (known as the measuring mass) is in uniform linear motion V in inertial axes OX_IY_I then the curvature of the path as seen from a set of axes OXY (Fig. 1) rotating with angular velocity Ω, is a direct measure of the rotation rate, being equivalent to that produced by a force $2V\Omega$ acting in OXY normal to V and Ω (Coriolis acceleration). Conversely, if a mass is constrained to travel in a straight line with velocity V in an axis system OXY rotating with angular velocity Ω relative to inertial axes OX_IY_I, then a force $2V\Omega$ must be applied normal to V and Ω to give it the requisite acceleration in OX_IY_I (Coriolis force).

The Coriolis effect is the basis of all inertial means of defining the direction of axes or of measuring their rotation, relative to inertial axes; in this lecture the term 'gyroscope' (6) has been used in this wider sense.

If the measuring mass is moving freely in inertial axes (free gyroscope), a measurement of rate is obtained if the path curvature ($V\Omega t^2/4$, Fig. 1) is measured at a fixed distance (free rate gyroscope). If, however, the mean direction of motion of the mass is continuously observed (OP, Fig. 1) the angle through which the direction of motion appears to turn in the rotating axis system is the angle through which the axes have turned (free directional gyroscope).

If the force required to make the measuring mass move with uniform linear motion in the rotating system is measured, the axis rotation rate is proportional to this force (constrained rate gyroscope).

In either case the measuring mass may have one degree of freedom (single-degree of freedom, or single-axis gyroscope) or two degrees of freedom (two-degree of freedom, or two-axis gyroscope).

In the most familiar form the motion of the measuring mass is rotary (the spinning gyroscope wheel), but linear or vibratory motion can also be used. The magnitude of the quantities involved in detecting axis rotation by the Coriolis effect are given in Table 1.

Measurement of external forces—the accelerometer

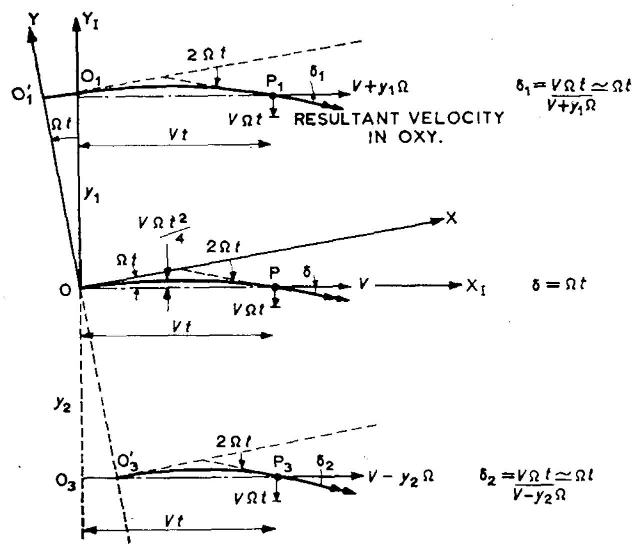
Fig. 2 shows, schematically, a vehicle acted on by gravitational force $\overline{G}$ and external specific forces* such as propulsion $\overline{P}$, lift $\overline{L}$, and drag $\overline{D}$, the vector sum of which determines the acceleration $\overline{A}_v$ in inertial axes. Carried inside the vehicle is a mass supported by a specific force $\overline{F}$ which can be measured by some instrumental means. The acceleration $\overline{A}_M$ of this mass is the vector sum of $\overline{F}$ and $\overline{G}$. Clearly if $\overline{A}_M = \overline{A}_v$ (and this can be determined within the vehicle), then $\overline{F} = \overline{P}+\overline{D}+\overline{L}$, i.e. the external applied forces have been measured.

If the gravitational vector $\overline{G}$ is known at all points then the acceleration of the vehicle can be determined in terms of components referred to inertial axes defined by gyroscopes in the vehicle.

Application of the basic measurements to navigation

The position of a body on the surface of the Earth is uniquely defined by the angular direction of the gravity vertical referred to a set of Earth reference axes with origin

* *Specific force is force per unit mass.*



Path	In inertial axes OX_IY_I Velocity	Acceleration	Curved path	In rotating axes OXY Deviation from straight line	Velocity	Acceleration
O_1P_1	V	0	O'_1P_1	$V\Omega t^2/4$	$V+y_1\Omega$	$2V\Omega$
OP	V	0	OP	$V\Omega t^2/4$	V	$2V\Omega$
O_3P_3	V	0	O'_3P_3	$V\Omega t^2/4$	$V-y_2\Omega$	$2V\Omega$

(To first order with $\frac{y\Omega}{V} \ll 1$)

Fig. 1. Transference of uniform linear motion in inertial axes into non-uniform motion in rotating axes (Coriolis force and acceleration)

Table 1. Coriolis effects of axis rotation relative to inertial axis

Motion of measuring mass			Coriolis effects				
1 Source of velocity	2 Velocity V in inertial axes, cm/s	3 Peak acceleration of motion in inertial axes $\dot{V}/g$ (g = 981 cm/s²)	4 Coriolis acceleration $2V\Omega$*, cm/s²	5 (Coriolis acceleration) $\frac{2V\Omega}{g}$	6 Ratio of peak Coriolis acceleration to peak acceleration of motion $\frac{2V\Omega}{\dot{V}} = \frac{2\Omega}{\omega}$	7 Deviation from a straight line $\frac{\Omega}{4V}$, cm/cm² of path†	8 Mean angular deviation $\frac{\Omega}{V}$, rad/cm of path‡
Linear motion	1×10^4	—	$1{\cdot}6 \times 10^{-3}$	$1{\cdot}6 \times 10^{-6}g$	—	$2{\cdot}0 \times 10^{-12}$	$8{\cdot}0 \times 10^{-12}$
Linear motion	3×10^{10} (velocity of light)§	—	$4{\cdot}8 \times 10^3$	$4{\cdot}9g$	—	$6{\cdot}7 \times 10^{-19}$	$2{\cdot}7 \times 10^{-18}$
Vibratory motion of 10^{-1} cm at 400 vibration/s ($\omega = 2{\cdot}5 \times 10^3$ rad/s)	250 peak	$635g$	4×10^{-5} peak	$4{\cdot}1 \times 10^{-8}g$ peak	$6{\cdot}4 \times 10^{-11}$	—	—
Wheel of diameter 4 cm rotating at ω = 400 rev/s	$5{\cdot}0 \times 10^3$ peak	$1{\cdot}3 \times 10^4 g$ at periphery	$8{\cdot}5 \times 10^{-4}$ peak	$8{\cdot}5 \times 10^{-7}g$ peak	$6{\cdot}4 \times 10^{-11}$	—	—
Wheel of diameter 8 cm rotating at ω = 400 rev/s	$1{\cdot}0 \times 10^4$ peak	$2{\cdot}6 \times 10^4 g$ at periphery	$1{\cdot}6 \times 10^{-3}$ peak	$1{\cdot}6 \times 10^{-6}g$ peak	$6{\cdot}4 \times 10^{-11}$	—	—

* Axis rotation rate $\Omega = 8 \times 10^{-8}$ rad/s, which is approximately equivalent to a rotation rate of one arc minute per hour or just over 10^{-3} of the Earth's rotation (1 milli-earth-rotation unit (meru)).
† The effective radius of an atom is 10^{-8} cm; electrical transducers can detect less than 10^{-9} cm displacement.
‡ The mean distance of the Sun from the Earth is $1{\cdot}5 \times 10^{13}$ cm; 10^{-12} rad is thus the angle subtended at the Earth by a length of 15 cm at the Sun.
§ By Einstein's general theory of relativity, propagation of light is only rectilinear in inertial axes, and gravity (or acceleration) introduces exactly the same curvature as would obtain with a free particle (equivalent mass $h\nu/c^2 = 3 \times 10^{-33}$ gm per photon for red light) travelling at the same velocity (7); this is the basis of recent gyroscope developments that will be discussed.

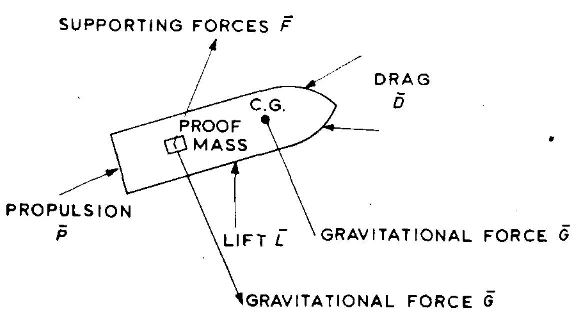
at the centre of the Earth—that is by latitude and longitude; the Earth reference axes used are related to the 'fixed' stars through sidereal time (Fig. 3).

The classical method of navigation by direct observation of the stars is subject to two difficulties:

(1) That of maintaining a true vertical on a moving vehicle, and

(2) That navigational information is lost when the stars are not visible.

By combining the gyroscope and accelerometer an indicated vertical independent of vehicle motion can be produced (with a characteristic 84 min Schuler period (3) (8)). The gyroscope, once aligned to a stellar datum, can maintain a memory of this direction (or of some datum rotating at a specified rate relative to it) and hence the angular relationship of the indicated vertical relative to



Force/unit mass for vehicle:

$(\bar{P}+\bar{D}+\bar{L})+$	$\bar{G}$	=	$\bar{A}_v$
External forces	Gravitational		Acceleration relative to inertial axes

Proof mass:

$\bar{F}$	+	$\bar{G}$	=	$\bar{A}_M$
Supporting forces		Gravitational		Acceleration relative to inertial axes

If $\bar{A}_M = \bar{A}_v$, $\bar{F} = \bar{P}+\bar{D}+\bar{L}$

Fig. 2. Measurement of external forces

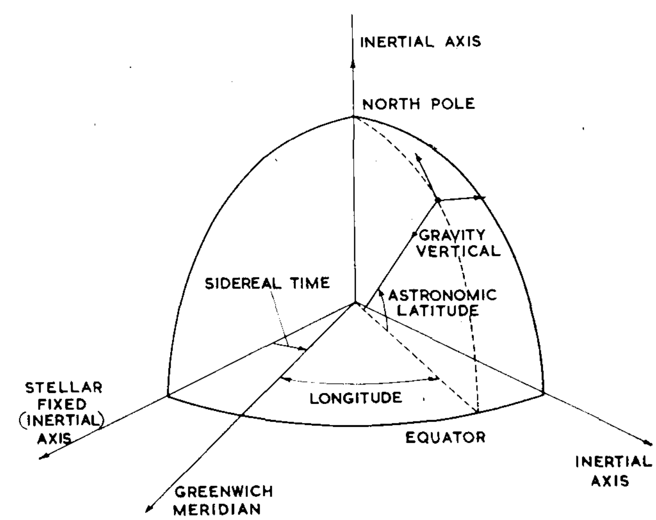


Fig. 3. Principles of terrestrial inertial navigation

the inertially maintained reference axes gives the position of the vehicle directly, for example, in terms of latitude and longitude.

This system concept is particularly valuable when a continuous knowledge of position is the main requirement, as for example in marine navigation. Typical accuracy aims are one nautical mile (n.m.) in ten hours.

An equally valid approach is to consider accelerometers measuring the external forces applied to the vehicle referred to inertial axes that are defined by gyroscopes on the vehicle. By integration the change in velocity due to the net external forces can then be determined. This system concept is particularly suited to ballistic missile and satellite operations.

For aircraft navigation both position and velocity are usually required, and this is obtained by using the Schuler tuned vertical as a direct source of velocity, with subsequent open loop integration to give position (3) (8). Typical accuracy aims are one nautical mile in one hour's flight.

Accuracy requirements for the inertial sensors

For navigation over the surface of the Earth an error of one arc minute in the indicated vertical, or the inertial reference with which it is compared, is equivalent (approximately) to one nautical mile position error, and a drift in either reference of one arc minute per hour is equivalent to a velocity error of one knot.

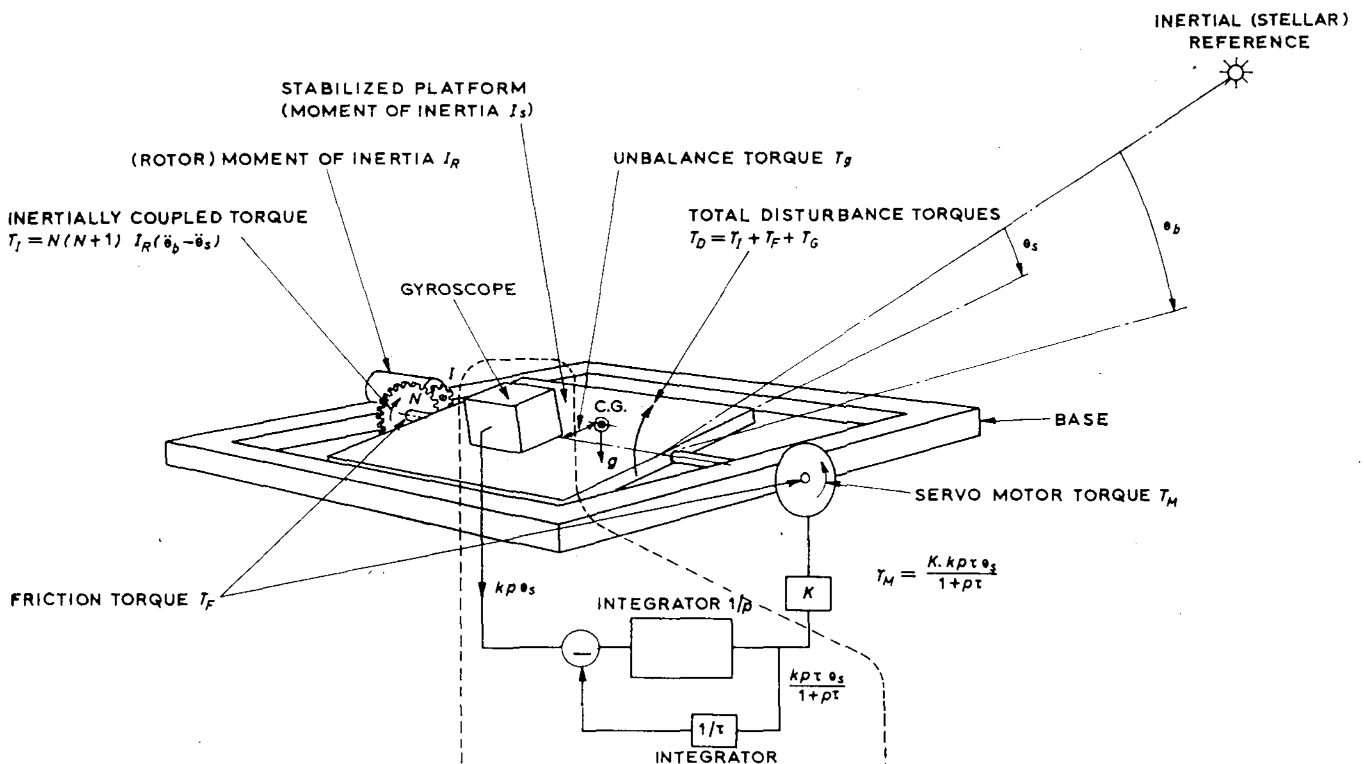
An error of one arc minute in the vertical would be produced by some spurious force in the accelerometer of $3\times10^{-4}g$. By comparison the Coriolis forces of axis rotation are (Table 1):

wheel of diameter 4 cm*, 1 arc minute/h, $8{\cdot}5\times10^{-7}g$
wheel of diameter 8 cm*, 1 arc minute in 10 h, $1{\cdot}6\times10^{-7}g$.

For aircraft and marine operation, the specific forces that have to be detected by the gyroscope are thus of the order 10^{-3} of those that have to be detected in the accelerometer, to achieve comparable contributions to navigation accuracy. Practical experience is in line with simple reasoning, the gyroscope problem being dominant for these applications.

For very long range high-speed vehicles, propelled for a short time, the accelerometer can, however, present the major problem.

* *These dimensions are illustrative only.*



Servo mechanism response:

$$I_s p^2 \theta_s = T_D - T_M$$

$$\theta_s = \frac{1}{Kk}\frac{1+p\tau}{p\tau}\frac{T_D}{\left(1+\frac{I_s p}{Kk\tau}+\frac{I_s p^2}{Kk}\right)}$$

Fig. 4. Gyro stabilized platform

THE USE OF THE GYROSCOPE IN AN INERTIAL SYSTEM

Special problems arise in maintaining the operational accuracy of the gyroscope as a directional reference for inertial navigation.

Servo stabilization of a platform

A gyroscope measures either rotation rate or angular rotation of an axis. In principle (**9**) it is possible to mount three rate measuring instruments and three accelerometers directly on the body of a vehicle and compute navigational outputs in any desired co-ordinate system, as long as the gravitational field is known at all points. Such systems are called 'gimbal-less' or 'strapped down' and pose the problem of measuring rates from, say, 1 arc minute per hour, to vehicle body rates which may be 180°/s or more (a range of 4×10^7 to 1). Unless very much lower body rotation rates obtain the problems are formidable, and the normal procedure in inertial navigation systems is to isolate the gyroscopes and accelerometers from body motions by a platform servo controlled from the gyroscope output (stabilized platform). The principle is shown (for one axis) in Fig. 4.

The platform is supported by bearings on the base, which is attached to the vehicle and may have large angular excursions (θ_b) relative to the inertial reference direction it is desired to define. Motion of the base is transmitted to the platform, for example, by bearing friction and by any inertial loads coupled through geared mechanisms; unbalance of the platform constitutes another component of the total disturbance torques T_D. The gyroscope either detects rotation θ_s of the platform directly (free gyroscope) or gives an output $k\,\mathrm{d}\theta_s/\mathrm{d}t$, which is integrated by some means to give angular displacement; in either case the measured or computed value of θ_s is applied with suitable gain K to an electrical torquing device that generates a torque T_M to oppose T_D. If the integrator is perfect, or the gyroscope perfectly free, any angular deflection θ_s will be 'remembered' indefinitely. In practice, however, instrumental imperfections may exist which cause θ_s to decay exponentially with a time constant τ.

The error θ_s in the definition of the inertial reference is given (in Laplace transform notation) in Fig. 4 as a function of the disturbance torque T_D. The second order term in the denominator determines the transient response (K may have to include time-dependent terms to ensure adequate stability) but the steady state error is determined by the $(1+p\tau)/p\tau$ term.

For a fixed disturbance torque T_D, θ_s thus tends to the value

$$\theta_s = \frac{T_D}{Kk}\left[1+\frac{t}{\tau}\right]$$

For maximum performance considerable importance therefore attaches to:

(1) The mechanical and electrical design of the platform and bearing assembly to keep all disturbance torques (T_D) to a minimum (high-torque direct-drive servo motors have been developed to avoid inertially coupled torques),

(2) As high a servo stiffness (Kk) as can be obtained consistent with adequate stability,

(3) An integration time constant (τ) that is long compared with the navigation period (t).

The need for isolation from base motion places considerable premium on small size and weight of inertial sensors. For example, an aircraft stabilized platform has to carry either two two-degree-of-freedom or three one-degree-of-freedom gyroscopes, and two or three accelerometers within a system of three or four gimbals; there is a considerable growth factor (of the order of 10 times) between weight of gyroscopes and inertial platform weight.

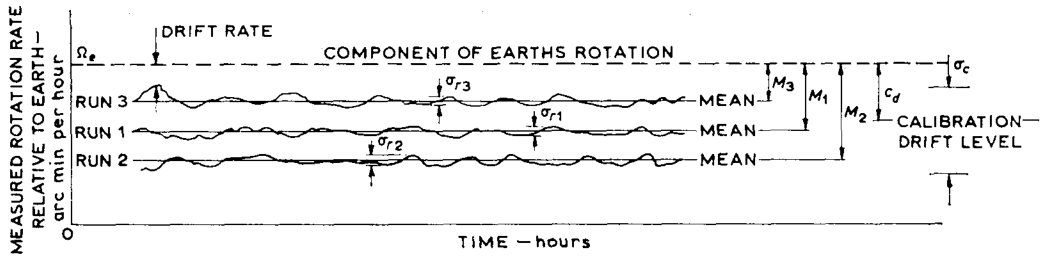
Consistency of gyroscope performance

Gyroscope 'drift' rate and calibration procedures

The order of accuracy required from a gyroscopic directional reference for inertial navigation is usually expressed in terms of a rate measurement (degrees/h, arc min/h, meru). Whether the gyroscope be free or restrained, the result of any parasitic force acting on the measuring mass will be indistinguishable from the Coriolis effect and will appear as a false measurement of axis rotation rate; this false rate is termed the 'drift rate'* or 'wander rate' of the gyroscope.

The user is concerned with the probable value of the drift rate which will obtain in use, and to which the navigational accuracy will be directly related. Characteristically gyroscope drift rate has a systematic component (Fig. 5) (which can be broken down (**3**) (**10**) into terms dependent on, and independent of, acceleration) and a so-called 'random drift' component. If truly random the latter is measured by the standard deviation, σ_r which enables the user to ascribe a confidence limit to the performance in use (95·4 per cent chance of being within $2\sigma_r$; 99·73 per cent chance of being within $3\sigma_r$, etc.). The near physical impossibility (so far) of reducing the systematic components of drift to zero by mechanical adjustment has meant that compensation has to be introduced by external means (electrical torque motor) for the value measured in some calibration procedure. Provided that equilibrium conditions of temperature, etc., have been established (**3**) (**10**), deviations from this calibration level then constitute the drift rate in use. The calibration of the gyroscope is, however, unfortunately

* *'Drift rate' is sometimes used to describe the angular variations in axis definition after a mean 'drift' component has been removed by some calibration process; the value of the 'drift' is then dependent on the method of calibration. 'Drift rate' in this lecture refers to any deviation of the gyroscopic reference from a prescribed rate relative to inertial axis. Gyroscopes are usually provided with an accurate means of applying a known force to the measuring mass so that the axis defined can be rotated (precessed) at a known rate relative to inertial axis, for example, to maintain a fixed attitude relative to the Earth. In this case the drift is referred to the precessed axis and includes any variations in the means of applying the precession.*



Standard deviation of drift rate from mean of any individual run:

$$\sigma_r^2 = \frac{\sigma_{r1}^2+\sigma_{r2}^2+\cdots+\sigma_{rn}^2}{n}$$

Calibration drift level:

$$c_d = \frac{M_1+M_2+\cdots+M_n}{n}$$

Standard deviation of means of individual runs from calibration level:

$$\sigma_c^2 = \frac{(c_D-M_1)^2+(c_D-M_2)^2+\cdots+(c_D-M_n)^2}{n-1}$$

Standard deviation from calibration drift level:

$$\sigma_d = \sqrt{(\sigma_c^2+\sigma_r^2)}$$

Fig. 5. Gyroscope calibration

subject to change, both through the method of usage (temperature fluctuations, vibration, shock, etc.), and over long periods of time (material creep and other factors). If recalibration before use is impractical (this usually imposes severe operational penalties) then the standard deviation, σ_c, of the calibration must also be determined during the calibration procedure by subjecting the gyroscope to the same environmental conditions as will obtain in use. The overall standard deviation of the drift in use is then given by

$$\sigma_d^2 = \sigma_c^2+\sigma_r^2$$

For most current designs (unless the gyroscope is operated continuously under very good environmental conditions) σ_c is very much greater than σ_r.

The methods of measurement of drift rate have been described by Cawood (**10**), and the most accurate is to time the rotation of the gyroscopically defined reference relative to the Earth.

It will be noted (Fig. 5) that the orientation of the gyroscope relative to the meridian and the vertical has to be known for the Earth reference rate Ω_e to be calculated. To separate the systematic component into gravitational and non-gravitational dependent terms the calibration has to be performed at different attitudes relative to gravity. Sufficient tests in every case have to be made to enable statistically significant results to be obtained.

With the need to determine also the variation in calibration with environment and method of use, calibration procedures become elaborate and prolonged if a high confidence is to be obtained in the performance in use. The calibration must be repeated at intervals, to maintain the required performance. Ultimately performance will become so erratic that complete overhaul of the gyroscope is required.

The cost to the user of maintaining performance

The economics of the use of inertial navigation are complex. For certain applications, for example in the military field, the operational advantages may be more significant than capital or maintenance cost.

Recent papers (**3**) (**11**) examining the application to civil air transport have shown that there is a wide spectrum of application of 'inertial navigation' techniques, both to navigation and flight control, with a complex interrelation between application, performance, and cost.

In examining the economic and engineering aspects of introducing inertial navigation equipment into airline operation, Sullings (**4**) shows the dominant effect of the gyroscope on the maintenance and overhaul cost. Using the same operational basis the annual cost of calibration and overhaul has been calculated as a function of the calibration and overhaul interval (Fig. 6).

The following assumptions have been made:

(1) Annual aircraft utilization—3500 hours.

(2) Turn-round time for repairing unserviceable equipment—6 weeks.

(3) If gyroscopes fail a functional performance test (i.e. do not maintain their calibration) the inertial platform is treated as unserviceable.

(4) Calibration cost is 25 per cent of the total production test cost, i.e. about 3 per cent of gyro cost*.

(5) The total cost of the gyroscopes is 40 per cent of the inertial platform cost (**3**).

(6) The overhaul cost is 65 per cent of the gyro cost*.

While for recalibration the turn-round time (which determines the cost of providing spare units) contributes 90 per cent of the total, for overhaul, the replacement cost

* *See section under heading 'The manufacturing cycle', p. 1143.*

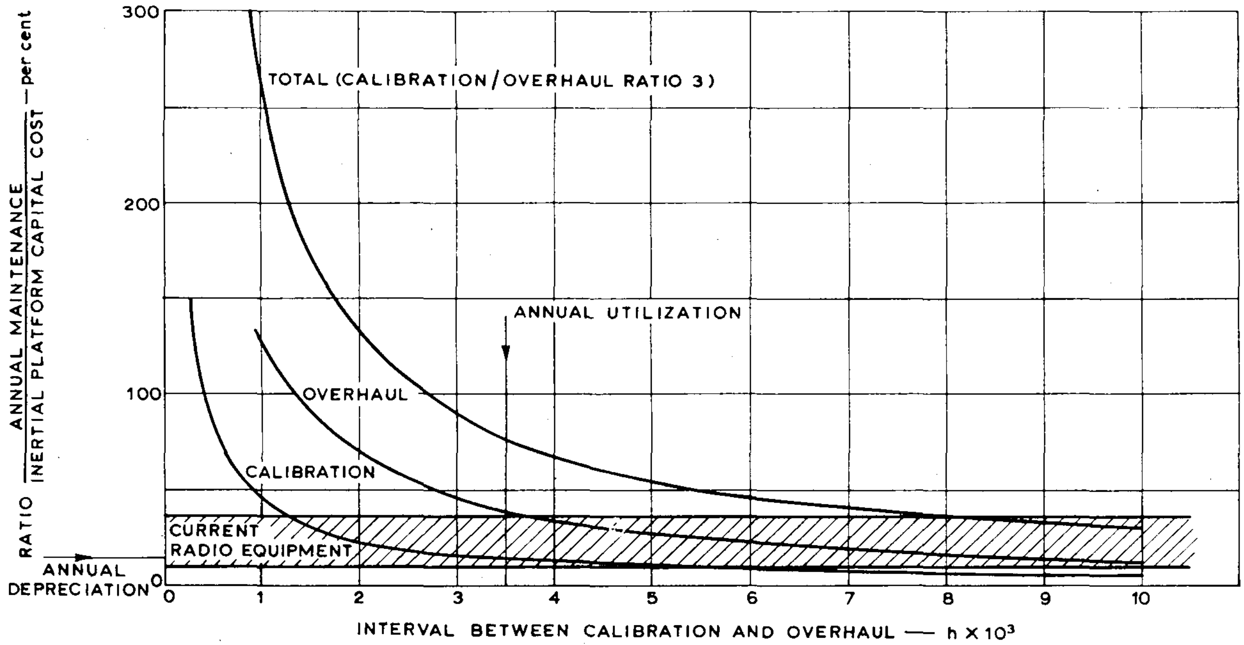


Fig. 6. Gyroscope contribution to annual maintenance costs (based on (4))

of the gyroscopes dominates (70 per cent). Fig. 6 shows, in addition to the breakdown into calibration and overhaul, the total annual cost as a function of overhaul interval on the assumption of three calibrations between each overhaul. The unit in each case is the capital cost of the inertial navigation platform itself (3) (excluding the navigation computer).

The equivalent maintenance costs of electronic equipment currently in airline use is also given (12). In considering the undoubted significance of the gyroscope performance in determining cost it must not be forgotten that an electronic failure could equally incur nearly the same costs as recalibration, and the high overall cost of the equipment places considerable onus on sound electrical and electronic design.

In considering the economics of use, Williams (5) has suggested that for improved equipment to be interesting to the operator, the return on investment from reduced aircraft operating costs should be at least 100 per cent after allowance for depreciation to zero and maintenance costs. With (say) a seven-year life the annual depreciation is under 15 per cent and hence in Fig. 6 the overwhelming significance of the (gyroscope) maintenance costs is apparent.

At a capital cost of, say, £25,000 for the inertial platform the 100 per cent level of Fig. 6 is equivalent to a direct operating cost of £7 per hour. Table 2 gives typical

Table 2. Relation to operating speed of typical average cost per flying hour for fuel and aircraft maintenance

	Maintenance and fuel cost/h, £	Flight time equivalent to £14 operating cost, min
Current subsonic jet .	270	3·1
Mach 2·2 cruise .	475	1·8
Mach 3 cruise .	610	1·4

average cost per flying hour for fuel and aircraft maintenance (over a 3000 n.m. stage length), related to operating speed. The chance of inertial navigation 'paying its way' by saving flight time (through improved navigation and flight control accuracy) thus increases with the speed of flight*.

As far as the gyro designer is concerned, however, viewed against the expected increase in serviceability of electronic equipment, a minimum design target would appear to be:

Overhaul life > 10 000 hours or 3 years,
Calibration life > 3500 hours or 12 months.

* *The actual saving is a complex function of distance flown, fuel consumption rate, meteorological conditions, etc., and the figures quoted should not be directly interpreted as estimates of probable cost saving.*

Part II: A brief review of some possible types

The importance to inertial navigation of the gyroscope performance, and the difficult nature of the problem, have attracted a wide variety of attempts to produce the 'ideal' gyroscope. Nevertheless, so far, large-scale production has been limited to a fairly narrow field from the many possibilities, although there is considerable variation in the individual technology used in implementing the basic design principle. To place present-day techniques in perspective a brief account will be given of the current field of possibilities. In doing so comparison will be made with the general requirements established in Part I.

The magnitudes of the allowable parasitic forces are so small that it is not surprising that obtaining adequate performance from gyroscopes for inertial navigation is inexorably linked with the design problem of maintaining the parasitic forces, and hence the drift rate, at an acceptable low level.

For example, parasitic forces arise from both gravitational and non-gravitational effects. The gravitational force is usually very large relative to the Coriolis force (Table 1) and there is no means of shielding against it; some inherent method of balancing is thus required within the gyroscope. Basically, this depends on the fact that if the direction of motion of a mass is reversed, the direction of the Coriolis force is also reversed relative to the gravitational force, and by comparative measurements on masses travelling in different directions the Coriolis force can be detected, in the presence of a large gravitational force.

The accuracy of balance that has to be achieved is one of the basic factors to be examined in comparing methods.

The measuring mass has also to be given a velocity; unless a stream of free particles is used, large acceleration forces (Table 1) will exist owing to the change in direction of the measuring mass, and a driving force may have to be applied to maintain a constant speed, i.e. counteract damping. The very large acceleration forces have to be kept to an acceptable level by dynamic balancing and special design measures are required to minimize driving forces. The accuracy with which this can be achieved can be a dominant factor in some methods.

Other forms of parasitic force (magnetic, electrostatic, elastic, frictional, fluid motion, etc.) have to be reduced by careful design, and the nature and likely magnitude of such forces have to be taken into account in comparing different methods.

A word of caution is, however, necessary about such basic comparisons. The relatively short history of the development of gyroscopes of inertial navigation quality shows that there is no magic road to success, the ultimate performance requirements only being reached by patient and persistent attention to every detail of engineering design over a long development period.

A realization of the basic problems that have to be solved does, however, enable an estimate to be made of the particular degree of engineering refinement required for a new approach to succeed, less profitable lines to be eliminated, and investigation to be concentrated on the major problems.

The general field to be surveyed is shown in Table 3.

Linear motion

It follows directly from previous statements on the measurement of axis rotation that if a stream of freely moving particles flows from A to B (Fig. 7) and another stream from B to A, then the separation $\frac{\Omega}{4}\left(\frac{l_1^2}{V_1}+\frac{l_2^2}{V_2}\right)$ of their paths is a direct measure of axis rotation rate Ω. Conversely, the measure of the force that has to be applied to reduce the separation to zero could be used.

The principle of balancing out gravitational force and the degree of accuracy required is apparent from Fig. 7. If there is a slight difference ΔV in the velocities of the two streams and a difference Δl in the path lengths then the drift rate is

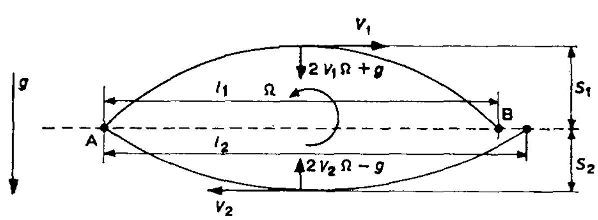
$$\omega_d = \frac{g}{2V}\left\{\frac{\Delta V}{V}+\frac{\Delta l}{l}\right\}$$

Mechanical application

From Table 1 column 5 it will be seen that for velocities of the order of 10^4 cm/s, if an accuracy of 1 min arc per hour is to be achieved in a $1g$ field, the velocities and path lengths must be matched to about one part in a million. Apart from the obvious difficulty of detecting the small displacement ($\sim 4\times10^{-10}$ cm for a path of 10 cm—Table 1 column 7) a mechanical gyroscope operating on this principle (for example, by an endless belt (6)) is thus unlikely to meet the accuracy requirements.

Charged particles

Proposals can be made for the use of charged particles as the active element; as 10 kilovolts accelerates an electron



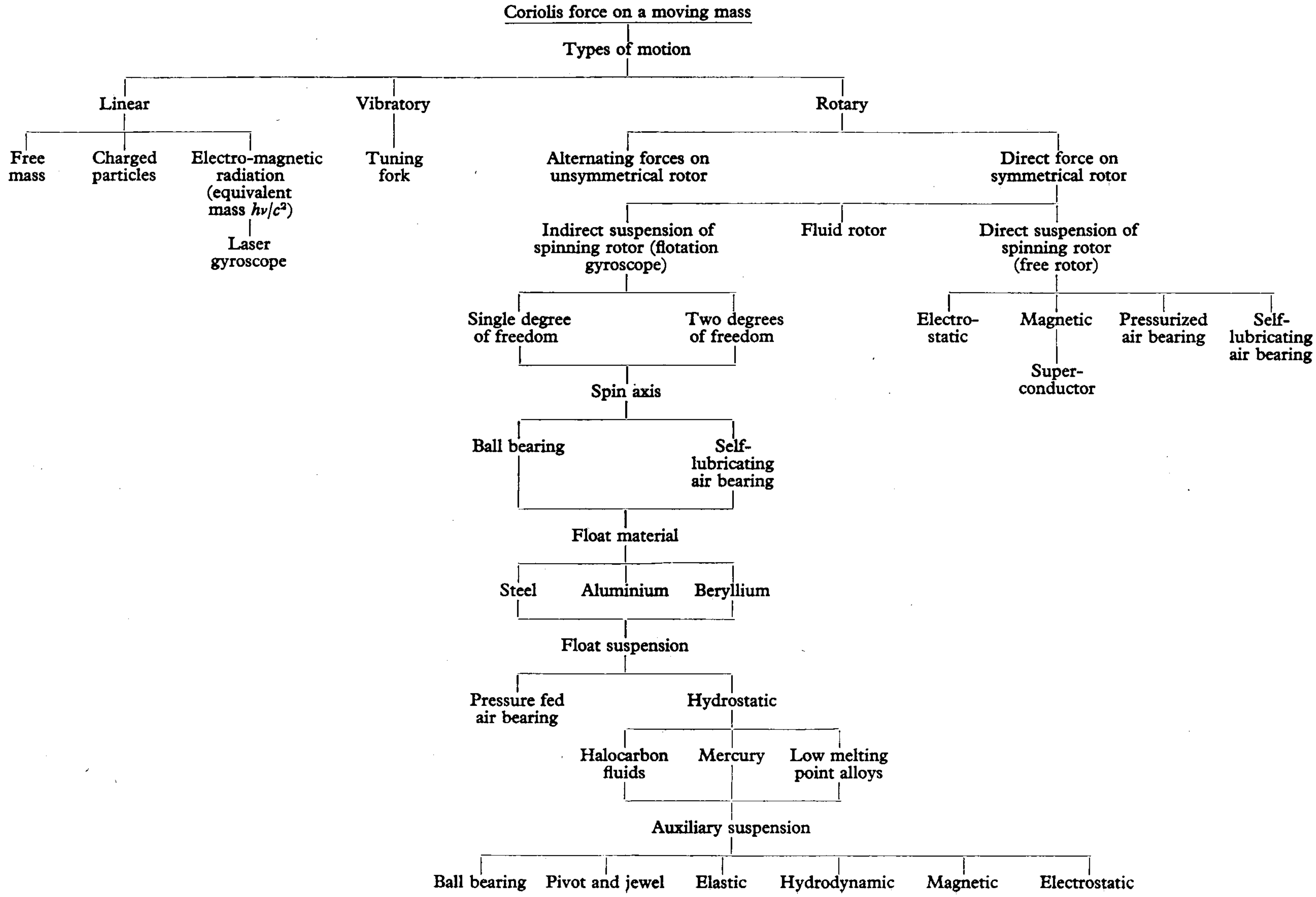
$$S_1 = \frac{\Omega l_1^2}{4V_1}\left[1+\frac{g}{2V_1\Omega}\right]$$

$$S_2 = \frac{\Omega l_2^2}{4V_2}\left[1-\frac{g}{2V_2\Omega}\right]$$

$$S_1+S_2 \simeq \frac{\Omega l^2}{2V}\left[1+\frac{g}{2V\Omega}\left(\frac{\Delta l}{l}+\frac{\Delta V}{V}\right)\right]$$

Fig. 7. Gravitational effect in linear gyroscope

Table 3. Some possible types of gyroscope



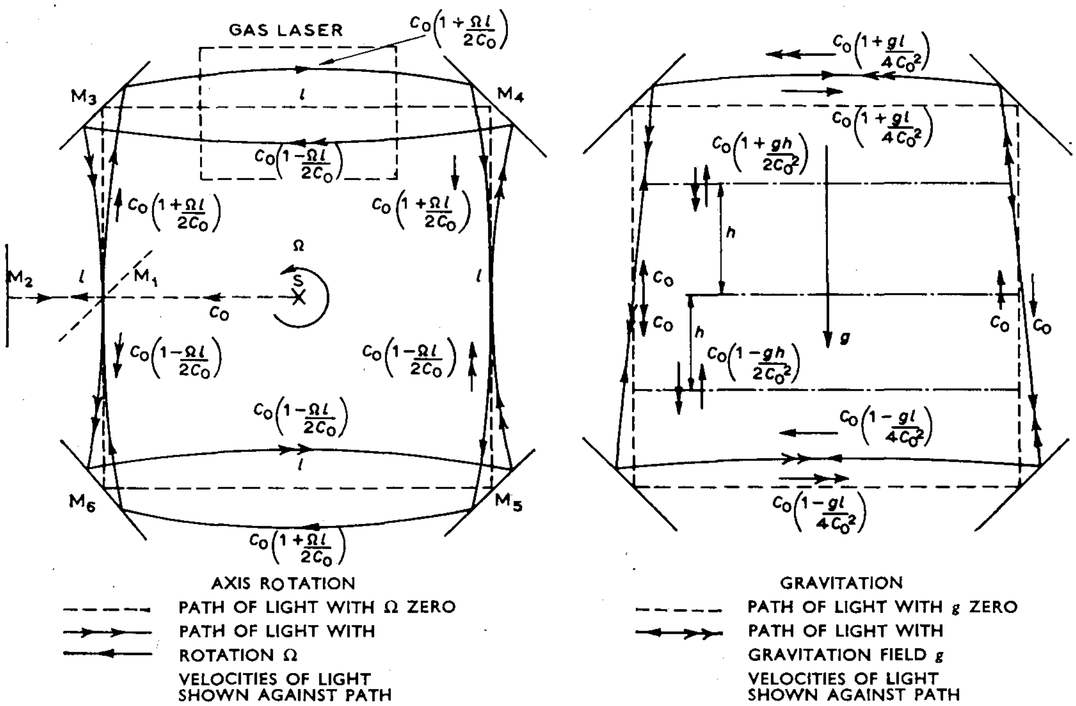


Fig. 8. Principle of the laser gyroscope

to 20 per cent of the velocity of light, high velocities are readily obtainable and the effect of gravity is correspondingly reduced (Table 1 column 5). A charged particle moving in a magnetic field is, however, subject to a force proportional to the velocity and which changes direction with direction of motion; the force due to the magnetic field is thus indistinguishable from Coriolis force. The ratio of the magnetic force to the Coriolis force is $He/2m\Omega$, which has the value $1{\cdot}1\times10^{14}$ per gauss ($\Omega=1$ meru). This adverse factor can be reduced somewhat (by a factor 10^4 to 10^5) by using heavy ions, but at the expense of velocity (if the accelerating potential is left constant). It is, however, a very large factor indeed to be eliminated entirely by screening and the increased speed has reduced the deviation by a corresponding factor.

Light (the laser gyroscope)

The most interesting, and most recent, application of linear motion is to use light as the active element.

If a reference frame is postulated in which light moves in a straight line with uniform velocity, then as seen from a rotating axis frame, the path will be curved in exactly the same manner as for the dynamic particles of Fig. 1. By the general theory of relativity the reference frame for linear propagation of light is the same as the inertial reference frame to which mechanical motion is referred (13).

Considering the motion of beams of light starting simultaneously from points O_1, O_2, O_3, in a rotating axes system (Fig. 1), these will arrive simultaneously at P_1, P, P_3, where $S = OP = O_1P_1 = O_3P_3 = C_0t$. As seen in the rotating system, however, the beams of light follow the curved paths OP, O'_1P_1, O'_3P_3, which are unequal in length. It can be shown that the effect of the curvature on the path length is second order in $\left(\frac{S\Omega}{C_0}\right)$; furthermore any changes in either time scales or length scales due to the Lorentz relativity contraction are also of second order in $(y\Omega/C_0)$.

Hence, to the first order of $y\Omega/C_0$ and $S\Omega/C_0$,

$$O'_1P_1 = S+y_1\Omega t = S\left(1+\frac{y_1\Omega}{C_0}\right) = C_0t\left(1+\frac{y_1\Omega}{C_0}\right)$$

$$OP = S = C_0t$$

$$O'_3P_3 = S-y_2\Omega t = S\left(1-\frac{y_2\Omega}{C_0}\right) = C_0t\left(1-\frac{y_2\Omega}{C_0}\right)$$

The velocity of the beam of light travelling with the direction of rotation is thus increased and that of the beam of light travelling against the direction of rotation is decreased, relative to a beam travelling radially from the centre of rotation.

An instrumental means of applying this principle to measuring rotation is illustrated in Fig. 8.

A beam of light from S is divided at mirrors M_1, M_2 into beams travelling with and against the direction of rotation. After reflection at the mirrors M_3–M_6 the two beams are recombined at M_1, M_2. The diagram shows the curved path of the light as seen in the rotating axis system. Neglecting terms in $(\Omega l/C_0)^2$ the actual path lengths from M_1 around the mirrors and back to M_1 are equal and the difference in transit time of the clockwise and anti-clockwise beams is

$$\Delta t = \frac{4l}{C_0}\left[\frac{1}{1-\frac{\Omega l}{2C_0}} - \frac{1}{1+\frac{\Omega l}{2C_0}}\right] \simeq \frac{4\Omega l^2}{C_0{}^2}$$

This can be detected by measuring the fringe shift

$$\Delta f = \frac{\Delta\phi}{2\pi} = \frac{C_0 \Delta t}{\lambda} = \frac{4\Omega l^2}{C_0 \lambda}$$

in the interference pattern produced when the two beams are recombined.

Although a terrestrial experiment to detect the rotation of the Earth by this means was hinted at by Oliver Lodge in 1897 (14) and a laboratory experiment to detect much greater rates carried out by Sagnac in 1913 (15), the first experimental detection of Earth rotation was not made until 1925, by Michelson and Gale (16), using a circuit of over 200 000 m^2 to obtain a fringe shift Δf of less than $\frac{1}{4}$.

Because of the very small effect ($\Delta f \sim 2\times 10^{-13}$/cm^2 for 10^{-3} of Earth's rate) no practical interest as a means of producing a gyroscope obtained until 1962 when the advent of laser light sources of very narrow spectral frequency bandwidths (i.e. of high phase stability) made possible detection of Earth's rotation within the relatively small area of a laboratory apparatus (17).

The method of detection used in the laser gyroscope is the conversion (by a gas laser, Fig. 8) of the phase difference around the path M_3–M_6 into a frequency difference between the two beams; counting the 'beats' then gives the angle through which the apparatus has been turned.

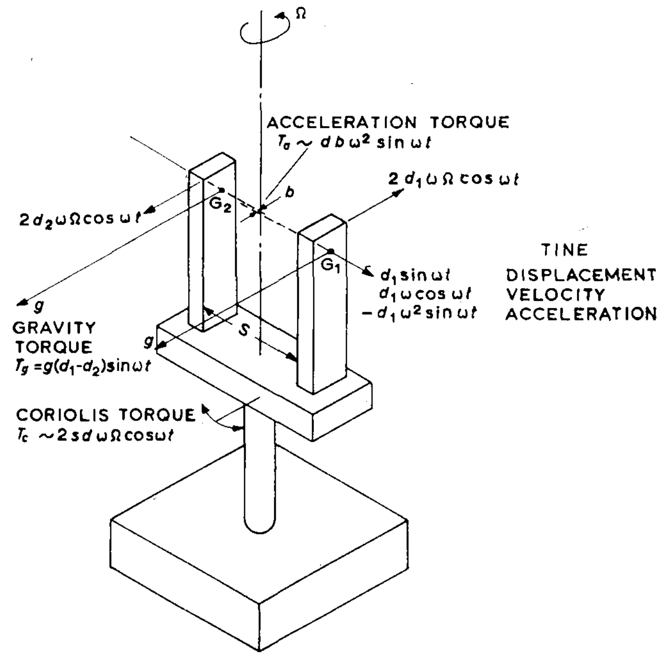
A change in velocity is also produced by gravity (7), and as shown in Fig. 8 the cancellation of gravitational effects depends on maintaining path equalities as in the linear gyroscope of Fig. 7. The ratio of the rotational to gravitational change in velocity is $\frac{2C_0\Omega}{g}$; this is equivalent to the Coriolis acceleration, to gravity, ratio for a particle of velocity C_0 (Table 1 column 5), and the high value of this ratio implies that a high degree of balance against gravity should readily be obtained.

The performance limitations of the laser gyroscope are, as yet, difficult to forecast, for the effect to be measured is so extremely small; whatever the instrumentation used the accuracy requirement is equivalent* to that of measuring the curvature of light rays under the Coriolis acceleration of the rotating axis frame.

The prospect of a gyroscope with a high degree of insensitivity to acceleration is, however, most attractive.

Vibratory motion

This form of gyroscope occurs in nature in the vibrating halters of the fly, which provide a directional stability reference; the principle was first applied as a gyroscopic reference by Meredith (18). The most familiar form is a tuning fork (Fig. 9) in which the alternating torque about the axis of the fork is detected by a torsion stem resonating at the fork frequency, the amplitude of torsional oscillation being measured by an electrical transducer. The instrument functions as a rate gyroscope with a response time determined by the resonant magnification of the torsion stem.



Ratio of Coriolis torque to parasitic torques:

$$\left[\frac{T_c}{T_a}\right] = \frac{s}{b}\frac{2\Omega}{\omega} \qquad \left[\frac{T_c}{T_g}\right] = \frac{s}{(d_1-d_2)}\frac{2d\omega\Omega}{g}$$

Fig. 9. Gyroscope using vibratory motion

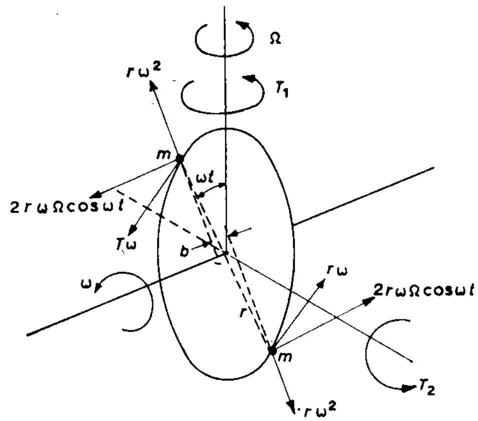
Parasitic torques are generated *inter alia* (Fig. 9) by unbalance in the acceleration forces arising from the tine vibration, and appear as a 'drift rate'. The drift rate has acceleration sensitive components (19), one cause of which is shown in Fig. 9, namely unequal tine vibration amplitude. These parasitic torques are in phase quadrature with the Coriolis torque, and discrimination can be introduced by phase-sensitive detection of the torsion transducer output. Frequency stability and other instrumental difficulties set a limit to the degree of discrimination that can be obtained, and a very high degree of mechanical balance has also to be maintained.

The ratio of Coriolis force to the vibratory forces is so small ($\sim 6\times 10^{-11}$, Table 1 column 6) that it is not surprising that 'navigational' accuracy has not, so far, been realized from this instrument. For less demanding requirements it has the advantages of very low power consumption, no rotating bearings to wear out, and the ability to measure a very large range of rotation rates.

Rotary motion

The Coriolis forces on diametrically opposite rotating

* *The curvature of a light beam can be considered to result from a change in velocity across the wave front (Fig. 1 and (7)), or vice versa.*



Coriolis torques:

$$T_1 = 2mr^2\omega\Omega \sin 2\omega t$$
$$T_2 = 2mr^2\omega\Omega(1+\cos 2\omega t)$$

Integrated over symmetrical rotor:

$$T_1 = 0 \quad T_2 = \frac{Mr^2}{2}\omega\Omega$$

Centrifugal torque:

$$T_1 = mbr\omega^2 \cos \omega t$$
$$T_2 = mbr\omega^2 \sin \omega t$$

Integrated over symmetrical rotor:

$$T_1 = T_2 = 0$$

Fig. 10. Gyroscope using rotary motion

masses constitute (Fig. 10) torques of $T_1 = 2mr^2\omega\Omega \sin 2\omega t$ about the axis of Ω, and $T_2 = 2mr^2\omega\Omega(1+\cos 2\omega t)$ about an axis normal to Ω and the spin axis.

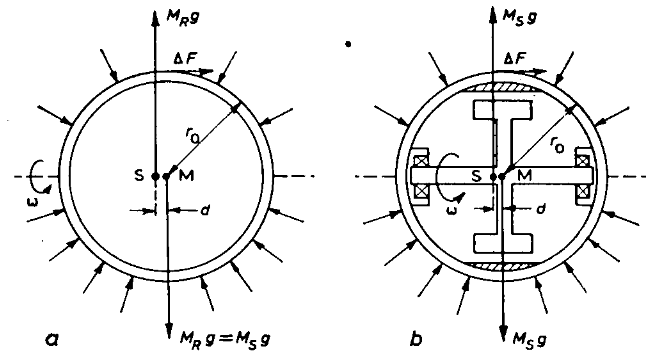
The (centrifugal) acceleration from the motion of the masses generates a parasitic torque about any axis normal to the spin axis, the magnitude being dependent on the dynamic unbalance. The ratio of the Coriolis acceleration to the motion acceleration has the same magnitude ($\sim 6\times 10^{-11}$, Table 1 column 6) as in the tuning fork. A very important difference obtains, however, namely the resulting torques are separated by an octave in frequency; instrumentally the discrimination that can thereby be obtained is of a much higher order than with the phase difference obtaining in the tuning fork.

In principle the alternating forces with frequency 2ω, which obtain with an asymmetrical rotor, can be used as a measure of axis rotation rate. While experimental gyroscopes operating on this principle have been made, the performance has not yet approached 'navigation' quality.

If a symmetrical rotor is used (i.e. a disc or wheel rotor) then by integration around the wheel the alternating components vanish, leaving as the direct component the familiar gyroscopic precession formula

$$T_2 = \frac{Mr^2\omega\Omega}{2} = I\omega\Omega$$

The unbalanced centrifugal torques then integrate to zero, provided that the system is linear. While a high degree of dynamic balance has to be obtained to guard against the possibility of non-linearities (20), balancing is no longer a first order design problem, as in the tuning fork.



a Direct rotor suspension ('free-rotor' gyroscope) *b* Indirect rotor suspension ('flotation' gyroscope)

Rotor mass M_R Sensitive element mass M_S

Centre of mass of sensitive element M Centre of suspension S

	(*a*)	(*b*)
Angular momentum	$\frac{2}{3} M_R r_0^2 \omega$	$M_R k^2 \omega$
A drift rate ω_d can be produced by:		
Torque (τ_d)	$\frac{2}{3} M_R r_0^2 \omega\omega_d$	$\frac{2}{3} M_R r_0^2 \omega\omega_d \left(\frac{3k^2}{2r_0^2}\right)$
Mass (ΔM_d) at radius r_0	$\frac{2M_R r_0 \omega\omega_d}{3g}$	$\frac{2M_R r_0 \omega\omega_d}{3g}\left(\frac{3k^2}{2r_0^2}\right)$
Rotor mass-centre instability (δ_R) relative to centre of suspension	$\frac{2r_0^2\omega\omega_d}{3g}$	$\frac{2r_0^2\omega\omega_d}{3g}\left(\frac{3k^2}{2r_0^2}\right)$
Sensitive element mass-centre instability (δ_S) relative to centre of suspension	$\frac{2r_0^2\omega\omega_d}{3g}$	$\frac{2r_0^2\omega\omega_d}{3g}\left(\frac{3M_R k^2}{2M_S r_0^2}\right)$
Ratio of tangential force (ΔF_d) to $1g$ suspension force	$\frac{2r_0\omega\omega_d}{3g}$	$\frac{2r_0\omega\omega_d}{3g}\left(\frac{3M_R k^2}{2M_S r_0^2}\right)$

Fig. 11. Suspension of spinning rotor

By far the greatest and the most successful effort has been directed towards the development of the symmetrical rotor gyroscope. Two basic types can be distinguished: a rotor directly supported by external forces (Fig. 11*a*), the free rotor gyroscope, and a rotor supported indirectly in a spin axis bearing mounted in a measuring assembly (Fig. 11*b*). In both cases the balancing of gravitational and acceleration effects is dependent on the accuracy of coincidence of the centre of suspension and the centre of mass of the whole assembly.

Table 4 shows the required constancy of torque level and suspension characteristics in relation to gyro parameters for typical direct and indirect suspension parameters.

Free rotor gyroscopes

One recently reported free rotor gyroscope (21) uses rotating fluid driven by spinning the spherical container; the time constant is, however, very low (0·1–1 second). Higher time constants can be obtained with a solid free rotor suspended on a self-lubricating spherical air bearing (22).

For maximum performance, however, an essentially free gyroscope of very long time constant is obtained by spinning a freely suspended spherical rotor and then allowing it to coast in a very high vacuum, coast-down times of several thousand hours being obtained (23).

The main source of spurious drift-inducing torques is

Table 4. Influence of gyroscope design characteristics on parasitic torque sensitivity

Gyro characteristics (at 400 rev/s; $\omega = 2{\cdot}5 \times 10^3$ rad/s)						Constancy of gyro characteristics for drift rate of 1 arc min/h ($\omega_d = 8 \times 10^{-8}$ rad/s)					
	Sensitive element					Torque level (τ_d)		Suspension characteristics			
1 Type of gyro	2 Radius r_0, cm	3 Mass M_S, g	4 Angular momentum $H = M_R k^2 \omega$, g cm² s⁻¹	5 $\frac{3k^2}{2r_0^2}$	6 $\frac{M_R}{M_S}$	7 $\tau_d = H\omega_d$ dyn cm	8 Mass at radius r_0 $\Delta M_d = \frac{H\omega_d}{r_0}$, μg	9 Stability of sensitive element mass centre $\delta_S = \frac{H\omega_d}{M_S g}$ micro-inches	9 (cont.) 10^{-8} cm	10 Stability of rotor mass centre $\delta_R = \frac{H\omega_d}{M_R g}$, 10^{-8} cm	11 Ratio of tangential force to suspension force (Under 1g) $\frac{\Delta F_d}{M_S g} = \frac{H\omega_d}{r_0 M_S g}$
1. Thin (0·2 cm) spherical shell free rotor (beryllium)	4	75	2×10^6	~0·95	1·0	0·16	40	0·8	210	210	$5{\cdot}3 \times 10^{-7}$
	2	19	$1{\cdot}3 \times 10^5$	~0·9	1·0	0·01	5	0·2	53	53	$2{\cdot}7 \times 10^{-7}$
2. Solid spherical free rotor (beryllium)	4	495	8×10^6	0·6	1·0	0·64	160	0·5	130	130	$3{\cdot}2 \times 10^{-7}$
3. Single degree of freedom halocarbon flotation—cylindrical steel float	4·1	900	6×10^6	0·87	0·27	0·5	130	0·22	55	200	$1{\cdot}3 \times 10^{-7}$
4. Single degree of freedom halocarbon flotation—aluminium float	1·8	70	10^5	0·76	0·32	0·008	4·5	0·045	11	35	$0{\cdot}6 \times 10^{-7}$
5. Single degree of freedom halocarbon flotation—beryllium float	2·2	125	5×10^5	0·85	0·6	0·04	18	0·13	33	53	$1{\cdot}5 \times 10^{-7}$

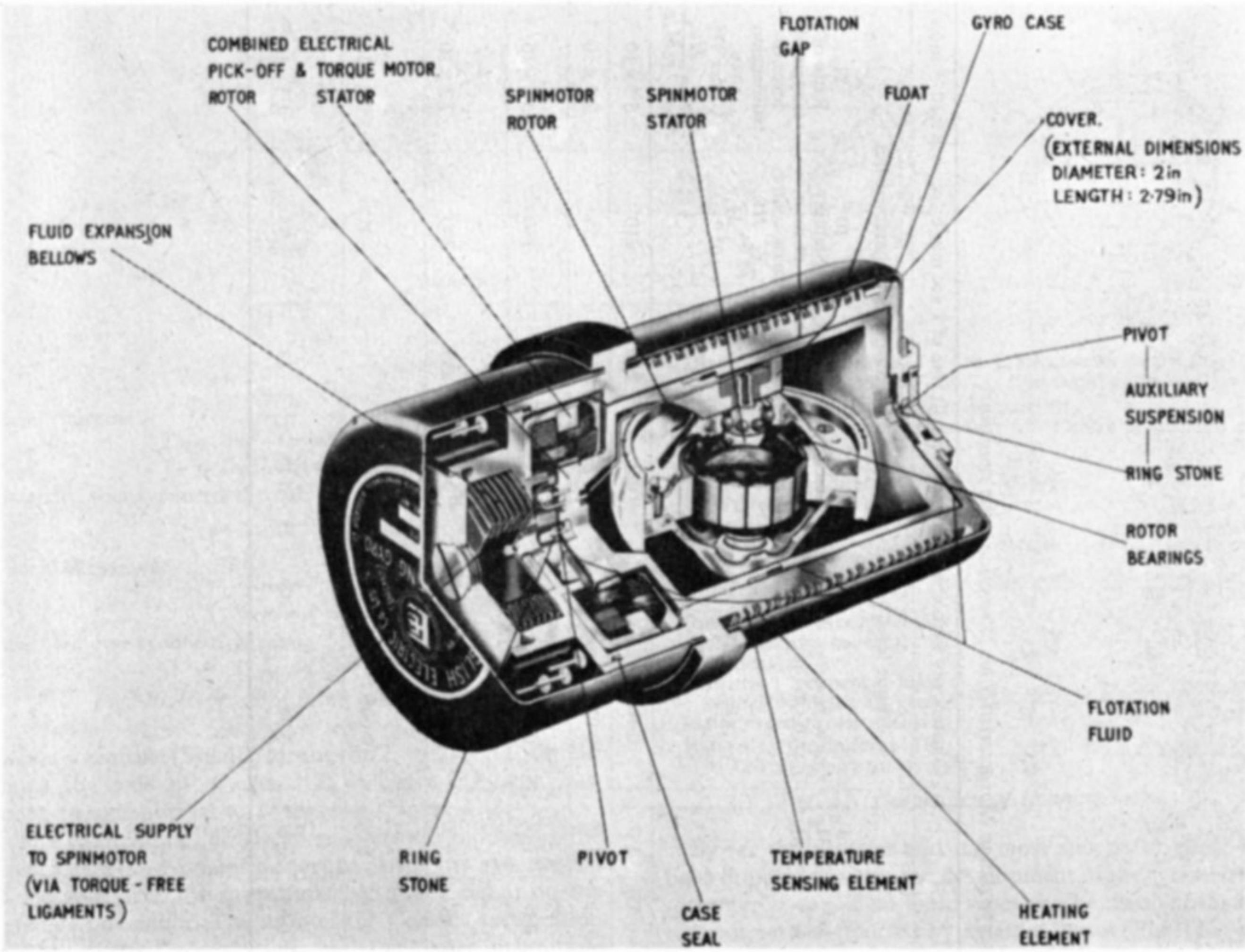


Fig. 12. A typical flotation gyroscope

(Courtesy of British Aircraft Corporation Ltd)

then the suspension. The design aim is to support a perfectly spherical rotor of uniform thickness by means of a force that is both normal to the surface and acts through the centre of mass to the accuracy limits indicated in Table 4.

Interest centres on electrostatic (2) and super-conducting magnetic suspension (23), which both have the natural physical property of generating a normal suspension force. Departures from sphericity (for example, by centrifugal force), and non-uniform thickness of the rotor, mainly determine the parasitic suspension torque level, and the constancy of calibration depends on their constancy.

Indirect suspension

The majority of inertial gyroscopes in manufacture today are of the indirect suspension type using either a pressure-fed hydrodynamic air bearing (24) or hydrostatic pressure (flotation) together with some auxiliary suspension to take forces arising from small deviations from neutral bouyancy (22) (25). The design of the hydrostatic flotation gyroscope* is considerably influenced by the need to achieve neutral buoyancy in a suitable fluid, the most common being a halocarbon (halogen substituted hydrocarbon) of density 1·8–2·4. Beryllium is now commonly used as a float material in order to obtain maximum structural rigidity within the required density†, together with good thermal conductivity and a coefficient of expansion that matches steel. It also gives an efficient use of the mass of the sensitive element (Table 4, column 6).

The use of hydrodynamic air 'flotation' enables a greater density of float to be supported, and a rigid steel structure can be used. Considerable investigation has gone into reducing turbine torques from the supporting air (24), and the design, which stems from German V2 developments, has been applied to a range of U.S. ballistic missiles (2).

* *The work of Dr C. S. Draper and his staff at the Massachusetts Institute of Technology in first bringing this concept to fruition is a most notable achievement in inertial navigation technology.*

† *Ratio of modulus of elasticity to density: steel 3·8, beryllium 23·2.*

Aircraft, submarine, and other ballistic missile inertial navigators currently use hydrostatically supported designs (2).

The many options open to the designer of a flotation gyroscope are shown in Table 3 and some of the merits of these are discussed in the literature (22) (24) (25) (26).

Table 4 shows the parasitic torque-level requirements for typical flotation gyro designs in relation to the free-rotor type. The smaller gyroscopes (1–5×10^5 g cm^2 s^{-1} angular momentum) are commonly used in aircraft systems; the larger (6–10×10^6 g cm^2 s^{-1} angular momentum) in marine systems, with a higher basic accuracy requirement. Fig. 12 shows a section view of a typical design of a small single-degree-of-freedom flotation gyroscope.

Viewed against the stability requirements it is a remarkable tribute to the skill of the research, development, and production engineers, and of the manufacturing operatives, that it is possible to obtain such a consistency in a production article.

Part III: Some aspects of obtaining consistent performance and long life from the flotation gyroscope

Flotation gyroscopes can now give the order of performance required for inertial navigation, but there is considerable scope for reducing both capital and maintenance costs (including calibration). The field surveyed in Part II is very wide and it is tempting to look to new methods to obtain these objectives.

Very large sums of money have, however, gone into research and development on the flotation gyroscope, particularly in the U.S.A., and U.S. designs have been put into production, under licence, in the United Kingdom. This accumulated experience cannot be lightly set aside, but it is very pertinent to examine whether further development is likely to yield an adequate return from investment.

As in other products the process of introducing a U.S. design into manufacture in the U.K. has not been a static one and developments from, and improvements to, the U.S. designs and manufacturing processes have taken place; the object of Part III of this paper is to present some of this work as a guide to what improvement potential exists.

Emphasis will be placed on experimental investigation that demonstrates the principles that have to be followed in design, development, and production, to obtain consistent performance and long life, rather than on individual merits of particular designs or processes.

THE MANUFACTURING CYCLE

The production factors that affect manufacturing and maintenance costs are illustrated by considering the manufacturing cycle of a typical flotation gyroscope as shown in Fig. 13. The small cost proportion of the raw material and bought out components will be noted. The machining and finishing cost includes a considerable amount of work in removing burrs, etc. from machined components; for this phase no adequate substitute has been found for individual

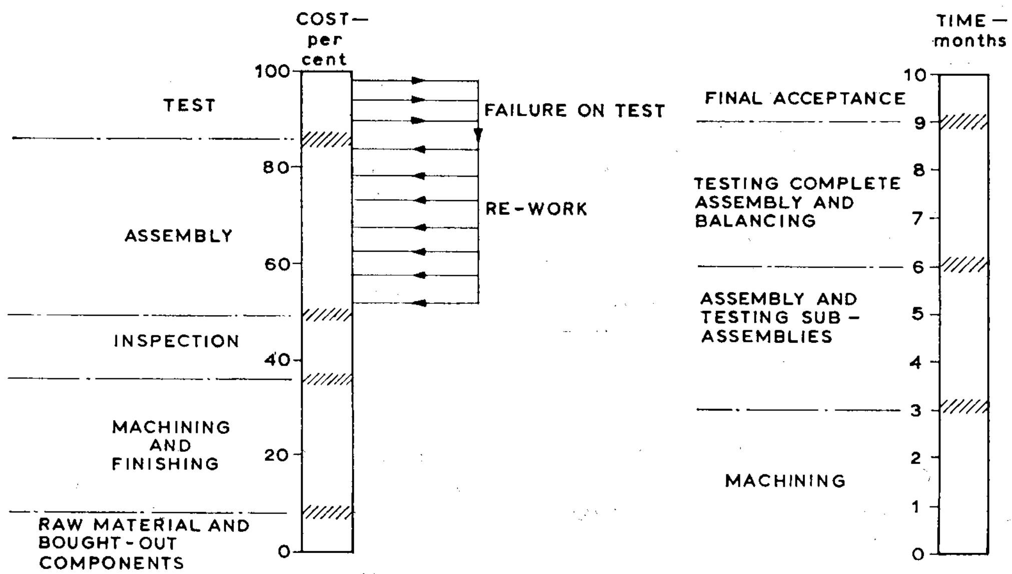
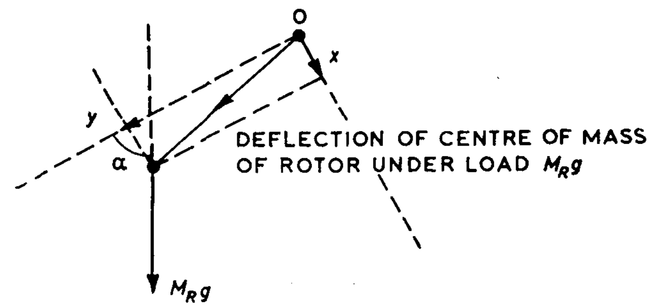


Fig. 13. Manufacturing cycle of flotation gyroscope



Compliance along Ox C_1 Compliance along Oy C_2

$$M_R g \cos\alpha\, C_2 = y \quad M_R g \sin\alpha\, C_1 = x$$

$$\text{Parasitic torque about O} = M_R g x \cos\alpha - M_R g y \sin\alpha = M_R{}^2 g^2 \sin\alpha \cos\alpha\,(C_2 - C_1)$$

Fig. 14. Aniso-elastic suspension torque

inspection under a microscope and removal by hand (dental tools are commonly used).

Gyroscopes that fail during the test procedure (which includes a series of tests for adjustment and balancing) are fed back into the appropriate part of the assembly line for reworking. The unit cost in Fig. 13 is the total cost to the customer, including an 'average' amount of rework. The extent of rework and the particular stage of assembly at which rejection occurs are major factors, however, in determining the capital cost of the gyroscope.

The cost of repairs of gyroscopes returned from the customer depends on the nature of the fault. If the float does not have to be stripped then the repair and recalibration may be 30–40 per cent of the unit cost. If, however, the bearings are faulty and the float has to be stripped, then the repair cost can rise to 60–70 per cent.

ROTOR (SPIN-AXIS) BEARINGS

The bearing performance requirement—to keep the mean position of centre of mass of the rotor fixed relative to the centre of suspension of the float to within a few tenths of a microinch (Table 4) is formidable, and it is not surprising that the rotor bearings are a major factor in obtaining consistent performance and adequate life.

Bearings have to be designed to reduce random mass shifts of both the rotor and the constituent parts of the bearing to a minimum, and the elastic deflection of the rotor (bearings and supporting structure) relative to the centre of suspension has to be in the direction of the applied force, otherwise a parasitic torque can be produced (aniso-elasticity, Fig. 14). The conditions required for iso-elasticity are usually in conflict with those for long bearing life.

Ball bearings

So far, the majority of gyroscope designs have used ball bearings for the rotor. The bearings are preloaded and the angle of contact selected (with allowance for compliance of the supporting structure) so that the deflection of the rotor is iso-elastic within the design range of applied acceleration.

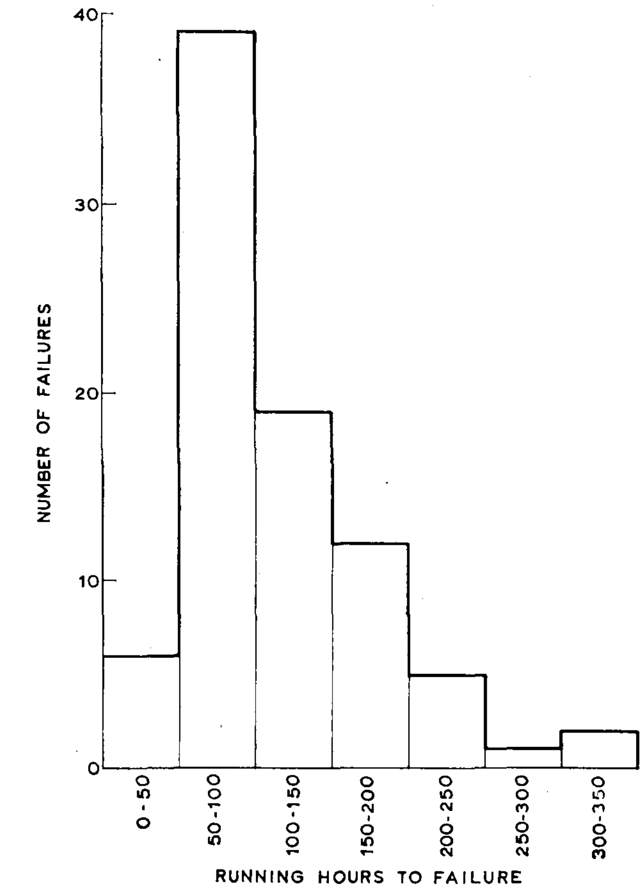


Fig. 15. Distribution of bearing failures

The problem of ball bearing life is graphically illustrated by Fig. 15, which shows the incidence of bearing failures encountered in the initial stages of introducing a U.S. design into U.K. production. Although U.S.-manufactured ball races were being used this failure rose to a sustained and serious level of 60 per cent of the bearings installed. Failure was manifest by excessive vibration and noise, large changes in bearing friction as indicated by the time for the rotor to run up or down in speed, and, in extreme conditions, inability to attain the synchronized running speed.

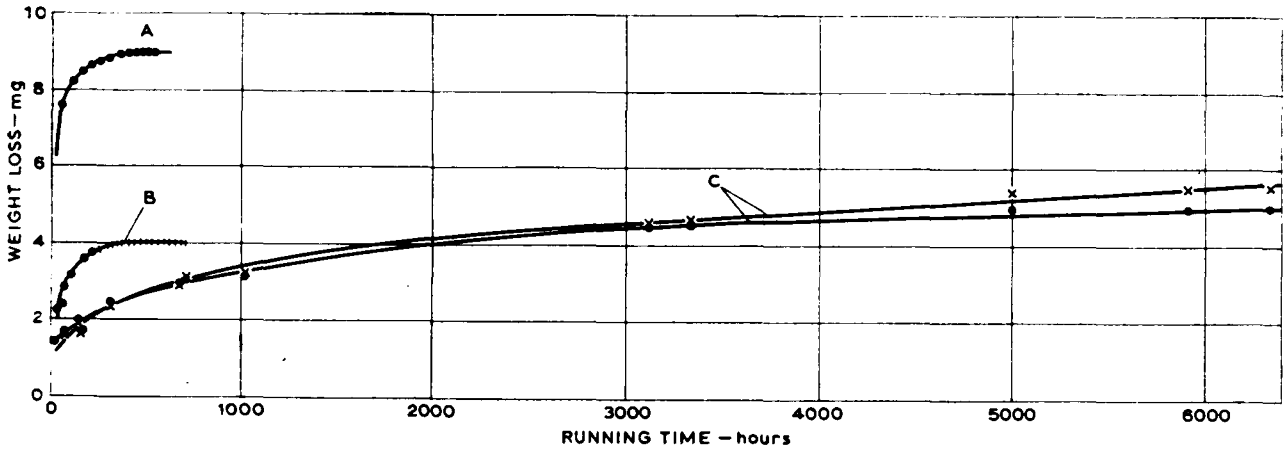
An experimental programme was conducted into three critical factors affecting bearing life:

(1) Lubrication,
(2) Presence of abrasive material, and
(3) Accuracy of manufacture.

Lubrication

Tests have been made* in which greased phenolic and oil-impregnated nylon ball retainers were spun, first on a spindle at operating speed and secondly on a special test housing with the balls installed on a complete bearing

* *By N. Sharp, T. Coldwell, and J. A. Carroll, Royal Aircraft Establishment.*



A Average curve of retainer greased by normal method with 12 mg of grease (10 mg of oil)—centrifugal effect only.
B Oil-impregnated (20 mg) sintered nylon—centrifugal and ball effects.
C Modified retainers incorporating grease pockets—centrifugal effect only.

Fig. 16. Loss of oil from ball-bearing retainers

assembly. The weight of lubricant lost, as a function of running hours, is shown in Fig. 16.

It is evident that in both cases (curves A and B) the oil metering rate is unsatisfactory for long running life, and that with the greased retainer practically all the lubricant had disappeared after a few hundred hours. Curve C shows the effect of modifying the retainer by introducing grease pockets to give a more desirable oil metering characteristic. This work on retainer design is continuing.

Although oil-impregnated nylon was adopted for this particular bearing design (the most satisfactory lubricating oil has been found to be a very cheap mineral oil), there is no universal solution to the lubrication problem. The effect of oil impregnation on performance stability (discussed under the heading 'systematic instabilities in performance') has to be considered, and a combination of high race accelerations, together with a high sensitivity to mass shifts, may make the use of oil impregnation unsatisfactory.

Abrasive material

The presence of residual traces of abrasive material on the bearing surfaces may have an adverse effect on bearing life. Quantitative measurements on the amount retained and on methods of cleaning have been made* using a small quantity of radioactive tracer (chromium 51) in the lapping abrasive (alumina). When the lapping compound was smeared on the surfaces without lapping, repeated rubbing of the surface with a cloth soaked in water or trichlorethylene removed the powder to the limit of detection (10^{-8} g). When the bearings had been lapped, however, an amount ranging between $1\cdot5\times10^{-8}$ g/cm^2 and $6\cdot5\times10^{-8}$ g/cm^2 was found to be retained.

The use of a chemical etchant was a prerequisite to removing the particles. Subsequent washing in solvents and ultrasonic cleaning, both commonly used cleaning methods, was ineffective in removing particles after etching; mechanical rubbing was the only effective way found of removing the particles.

The effect of the retained abrasive on ball race life is not known and special controlled tests are planned to investigate this.

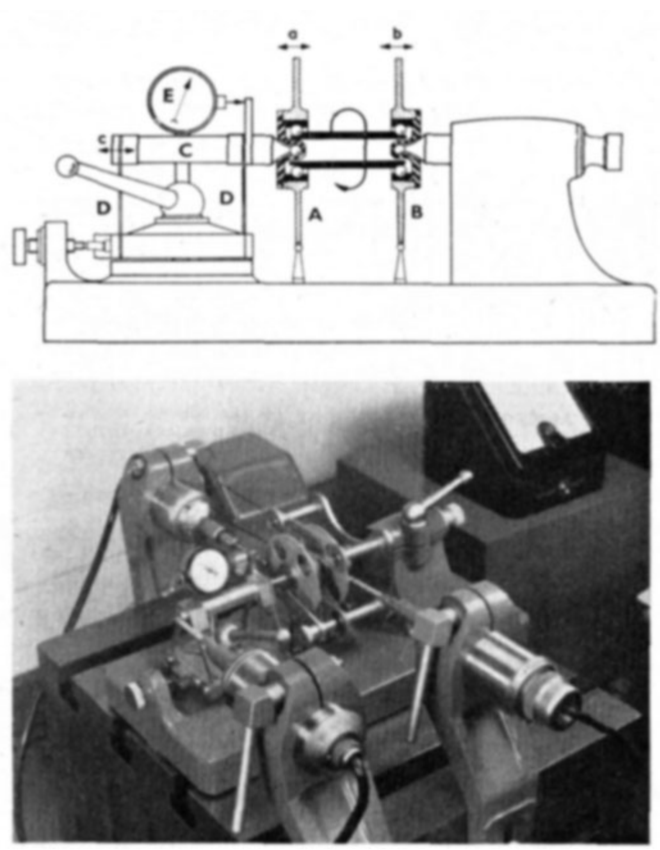


Fig. 17. Ball-bearing test machine

* *By R. A. Bacon and S. Pearson, Royal Aircraft Establishment.*

Accuracy of manufacture

The extent to which accuracy of bearing manufacture affects bearing life is difficult to quantify. The quality is sometimes assessed indirectly by measuring the bearing low-speed friction, or driving power at running speed, with attention to transient increases in power. Although useful, for example, in determining the torque reactions on the gyroscope sensitive element, these measurements are not directly applicable to manufacturing processes. Precise physical measurements can be made of the bearing elements, but in an assembly comprising some 14 balls, two pairs of inner and outer ball tracks, and a pair of drilled cages, this is tedious, and the overall assessment of bearing quality based on these data is difficult. A machine has therefore been developed* to assess bearing quality by producing records from a bearing assembly of the dimensional errors considered critical in gyroscope application.

* *By N. Sharp and R. A. Bacon, Royal Aircraft Establishment.*

The machine is shown in Fig. 17. The principle is to run a pair of bearings under the same preload conditions (provided by the leaf springs D) as obtains in use. Instead of the outer races being constrained they are supported in swash plates A and B and allowed to move freely in response to dimensional errors in the elements. The longitudinal motion (*c*) of the axial loading system and the lateral motions (*a*, *b*) of the swash plates are recorded by electro-magnetic gauge heads modified to give a probe load of only 0·5 g.

The inner bearing shaft is driven at low speed by a symmetrical lightweight belt.

Movement of a carrier plate caused by lack of square-

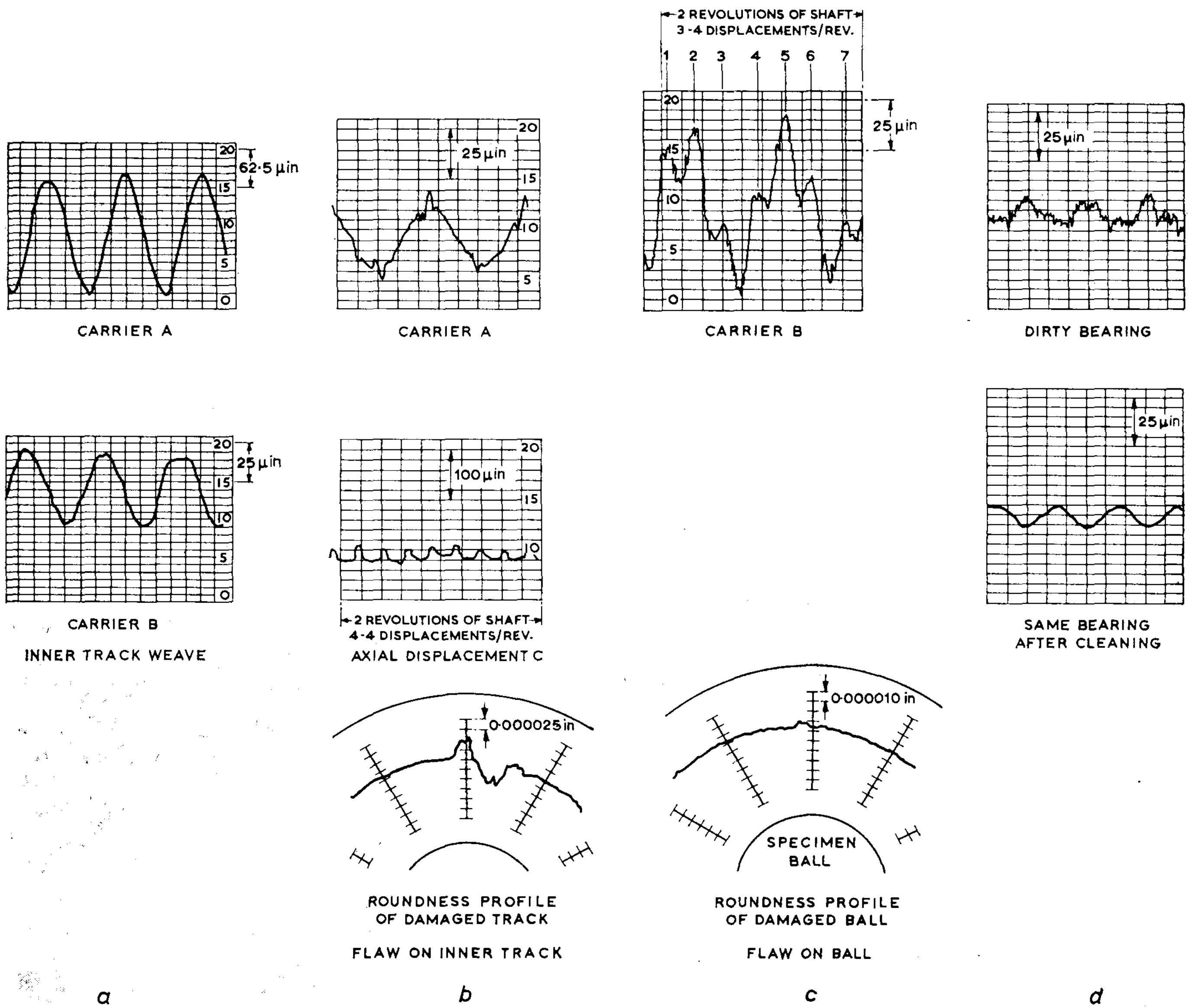


Fig. 18. Ball-race flaws detected by race-measuring machine

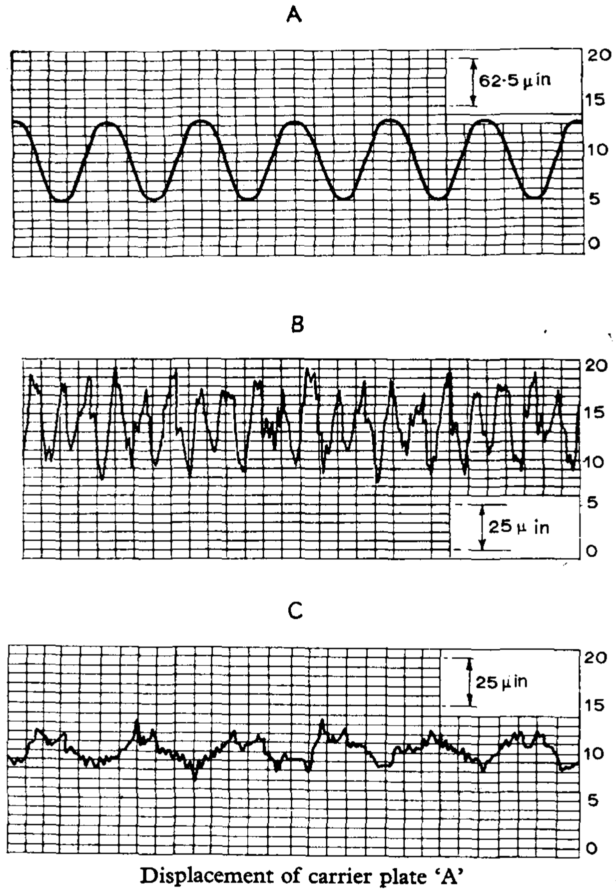


Fig. 19. Typical records from bearings supplied by three manufacturers

ness of the track and the axis results in a simple sinusoidal record, and this fault is apparent in the majority of the bearing assemblies tested (Fig. 18*a*). Deflections are caused by track and ball flows (Fig. 18*b*, *c*) contamination by dirt (Fig. 18*d*), etc. and can be specifically traced to source. An assessment of some 200 bearing assemblies showed considerable variation in the degree and type of manufacturing error present—outer ring displacements varied between 10 microinches and 300 microinches and axial displacements in the range 30–40 microinches; the maximum displacements corresponding (statically) to 40 per cent of the bearing preload. Strong 'family' characteristics were found on testing bearings from different manufacturers (Fig. 19), sufficient for the experienced observer to be able to identify the manufacturer from the record alone.

The next stage of the investigation is to try to establish correlation between the test records and performance in use, but in the meantime prudence has dictated the use of the machine to inspect all bearings before assembly and to reject bearings that exceed some arbitrary limit (for example, ±50 microinches 'weave' and ±10 microinches axial displacement).

Long-life ball bearings

The net result of the programme of investigation on ball bearings has been to reduce failures in production to the very low level of 1·6 per cent, measured over nearly 500 rotors. Recent results* on life tests of bearings also give grounds for encouragement that a life of over 10 000 hours may be achieved from a ball bearing supported rotor at a speed of 24 000 rev/min. Fig. 20*a* shows the measured run-down time of the rotor over a given speed range (1800–1200 rev/min) as a function of running time. A short run-down time signifies excessive friction, and a considerable increase signifies loss of preload due to wear. For comparison, Fig. 20*b* shows (on a very much extended timescale) a typical record from an earlier rotor assembly that failed after a short (500 h) running period.

Similar long lifetimes are reported by U.S. workers (**22**).

Gas bearings

The attractiveness of the unpressurized (self-pumping) hydrodynamic gas bearing in eliminating running wear is self-evident, and considerable effort has been, and is being, devoted to the design and development of gas spin bearings for gyroscope rotors. In addition to the greatly increased life of the bearing improved mass stability can be anticipated owing to the elimination of the individual elements in a ball race (and the lubricant).

Gas bearings, however, bring their own problems:

(1) Extremely high accuracy of manufacture is required, which results in high component costs and high ancillary equipment costs;

(2) High motor powers are required—the starting torque can be, say, five times greater than for an equivalent ball-bearing wheel and the running power 50 per cent greater;

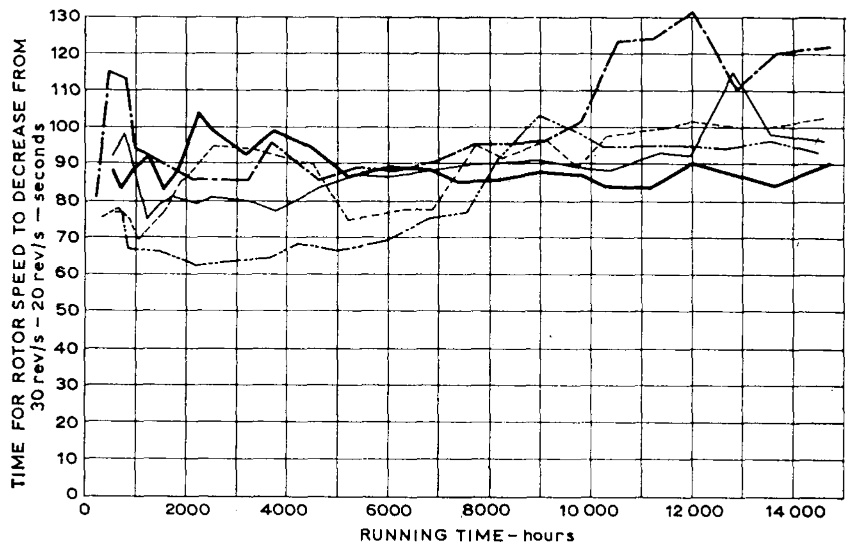
(3) During the starting and stopping of the wheel, metal-to-metal rubbing occurs which can cause rapid deterioration of the bearing and limits practical material selection to those materials that are abrasive resistant or self-lubricating;

(4) The requirement for iso-elastic construction causes considerable design, manufacturing and assembly problems; radial and axial compliances have to be matched by individual measurement and adjustment, and very small changes of bearing size or geometry will cause changes in these stiffnesses; the simplest (journal) bearing is also basically aniso-elastic (Fig. 21).

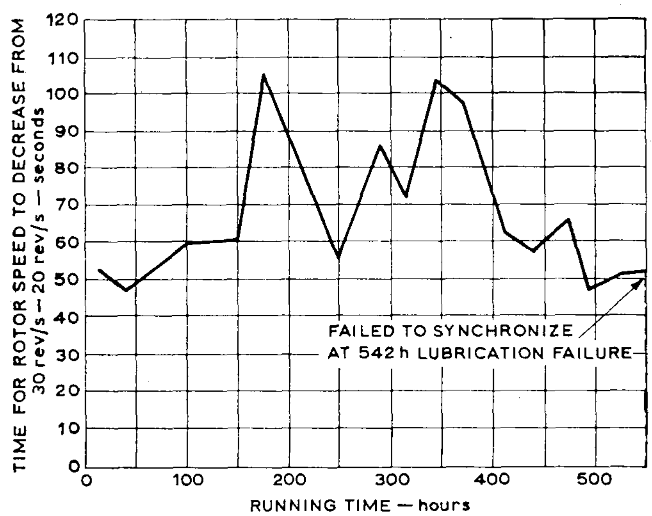
(5) Unloaded gas bearings are unstable at a lateral frequency of half the rotational speed; this vibration (whirl) is normally divergent, causing wheel destruction, and empirical design modifications are necessary to damp out the oscillation;

(6) The low vibration level due to absence of bearing noise may cause difficulties with friction in the auxiliary float suspension.

* *By Ferranti Ltd.*



a Rotors incorporating results of ball-bearing research programme (British bearings).



b Early gyro rotor.

Fig. 20. Gyroscope rotor life tests (ball bearings)

Fig. 22 shows a typical bearing design with cylindrical journal bearing and flat thrust plates incorporating Whipple spiral grooves.

The effect of bearing dimensions on the inception of whirl with an unloaded bearing is illustrated by Fig. 23.

STABILITY OF PERFORMANCE

Performance may deteriorate to below that specified, or calibration may change because of variations in parasitic torques with age. Many of the factors that cause long-term variations can also, in a more extreme form, be the cause of failure to meet performance requirements during manufacture or on final acceptance testing. Longer-term deterioration in performance arises mainly from:

(1) Float mass-balance instability due to spin axis bearing wear, and migration of lubricating oil within the float.

(2) Friction from the collection of contaminating particles in fine gaps between the float and the surrounding case, or from loads on the auxiliary suspension system due to the development of buoyancy errors.

(3) Changes in the null position of the motor electrical supply ligaments to the float because of hysteresis in the ligament material.

(4) Balance instability of the float due to the adherence of dirt or the formation of bubbles in the flotation fluid; bubbles may form by internal generation or because of the inefficiency of the casing seals.

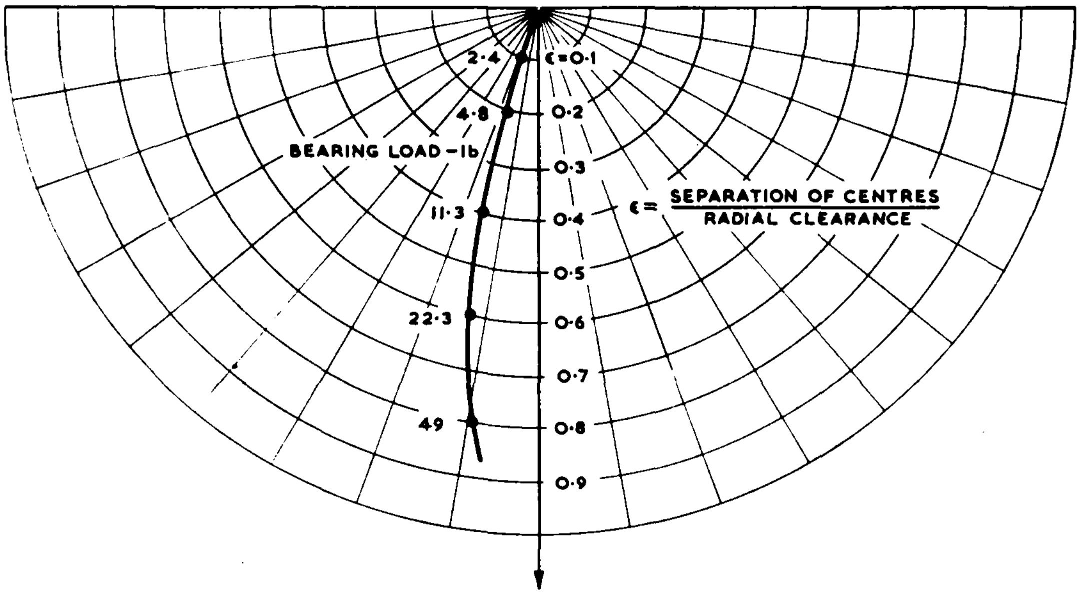
Contamination of the suspension fluid

General residue

Contamination is one of the major problems of gyroscope manufacture and considerable capital is invested in special facilities (27) for assembly under clean conditions. The cleaning procedures are extensive and much development has gone into perfecting them. Both solvent and aqueous cleaning are used, the former with ultrasonic cleaning and the latter with scrubbing (ultrasonic cleaning with aqueous solutions causes erosion). Solvents are filtered and a check made on the contamination by sample counts; as many as 12 consecutive ultrasonic cleanings with fresh fluid each time are used.

Analysis of residues shows that a very high proportion (as much as 90 per cent) is human residue (skin), the remainder being metallic (solder, etc.) and sand (silica). Skin finger-prints are particularly difficult to remove with solvents, and scrubbing is the only effective means.

Measurements on overall contamination have been made by extracting fluid from reject gyros and then stripping with careful flushing of the components; the particles are then counted. Fig. 24 shows the effect of improved cleaning and handling procedures. The large improvement obtained by attention to cleaning techniques is evident,



Bearing characteristics:

Length	1·25 in
Diameter	0·625 in
Radial clearance	120×10^{-6} in
Pressure	22 lbf in^{-2}
Speed	200 rev/s

Fig. 21. Aniso-elasticity of self-lubricating gas spin-axis bearing; locus of journal centre

(Courtesy of British Aircraft Corporation Ltd)

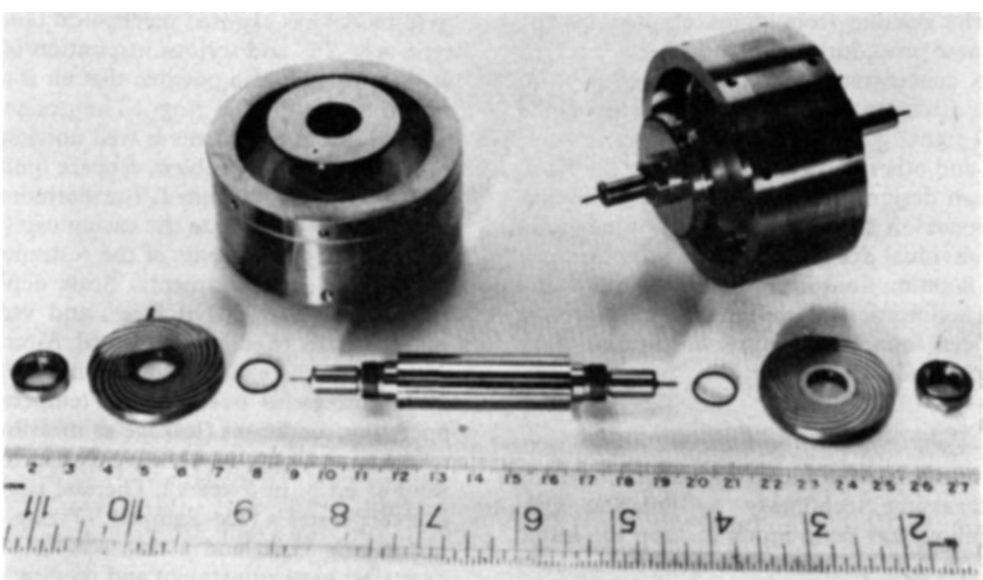


Fig. 22. Typical self-pumping gas bearing for gyroscope rotor

(Courtesy of British Aircraft Corporation Ltd)

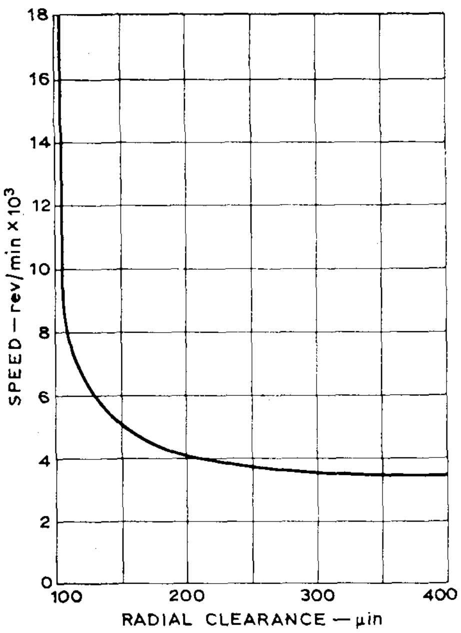
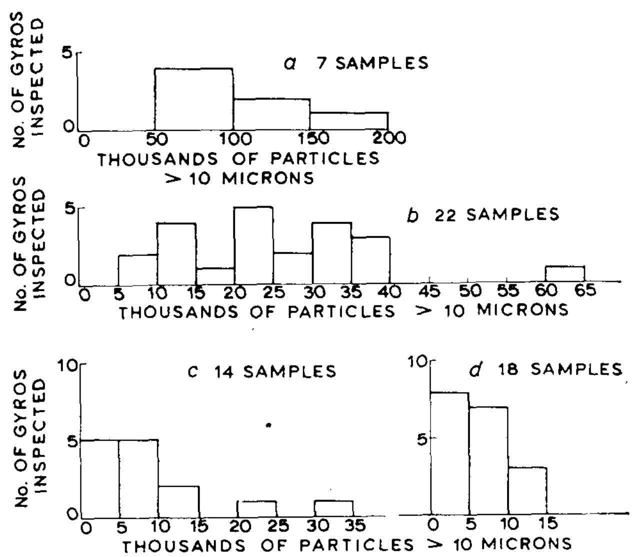


Fig. 23. Whirl inception speed against radial clearance (shaft vertical)

(Courtesy of British Aircraft Corporation Ltd)



a Original cleaning techniques.
b Improved cleaning techniques: 3 months' output before fitting sealing ring without handling.
c Improved cleaning techniques: first 3 months' output after fitting sealing ring without handling.
d Improved cleaning techniques: second 3 months' output after fitting sealing ring without handling.

Fig. 24. Influence of cleaning and assembly procedures on gyroscope fluid contamination

(Results by courtesy of Ferranti Ltd)

and also the effect of avoidance of handling components. The extent to which the human operator can cause contamination is emphasized by Fig. 25*c* which shows microphotographs of part of a filter through which solvent has been passed; previously (after the hands had been washed) finger tips were dipped for a few seconds into 2 litres of the solvent in a beaker. For comparison Fig. 25*a* and 25*b* shows the residue from gyros cleaned by the original and the new procedures of Fig. 24.

In addition to concentrating on cleaning, it pays to give considerable attention to the design of components such as electrical signal generators and torquing devices, balance weights, and other parts in contact with the fluid, to reduce, by clean design, the possibility of contamination. Direct evidence on the reduction in contamination resulting from individual design modifications to parts in contact with the flotation fluid is difficult to obtain, but a number of such modifications to designs manufactured in the U.K. have been found sufficiently worthwhile to be fed back into U.S. production.

Gas bubbles

The application in the U.K. of radio-valve-manufacturing vacuum and degassing technology to flotation gyro processing and filling, has done much to keep the initial gas content in a gyro to an acceptably low level. Fortunately the flotation fluids used will absorb a relatively large proportion of gas (13·5 per cent by volume at normal temperature and pressure (n.t.p.)) and this provides protection against gas leakage during life. As Fig. 26 shows, however, the gas content increases with life in a very variable manner (time constant of between 6 months and 5 years) the weak point being the casing seals. Fig. 27*b* shows a typical piston-type seal using a rubber 'O' ring. Work at the Royal Aircraft Establishment* using a glass gyro model has exposed mechanical failure of seals of this type at +2°C and serious inspiration of air past the seals at −20°C. It is also possible that air is admitted by diffusion through the 'O' rings. The design of effective seals for high vacuum systems is well understood, but the gyro presents a difficult problem of space limitation, and clamping forces are also limited. Furthermore, in some designs it is necessary to rotate the casing cap (without upsetting the seal) for adjustments of the restraints applied by gyro motor conductor ligaments. Seals depending on lapped metal faces, soft plated faces, and various intermediate gaskets were tried at the Royal Aircraft Establishment without complete success, but eventually one was developed to be successful over a wide temperature range at all operating conditions (leakage as measured by helium test, using a mass spectrometer, less than that to saturate the fluid at n.t.p. in 5 years). The seal is shown in Fig. 27*a*; it incorporates a face-clamped 'O' ring, as opposed to the piston-ring type, and a thin sealing band to be swaged over after gyro adjustment and proving trials: a temporary clamp (Fig. 27*c*) is used to allow adjustment prior to this final swaging. The design is much cleaner, and the chance of contamination of the fluid by the 'O' ring is reduced.

* *By M. Trapaud.*

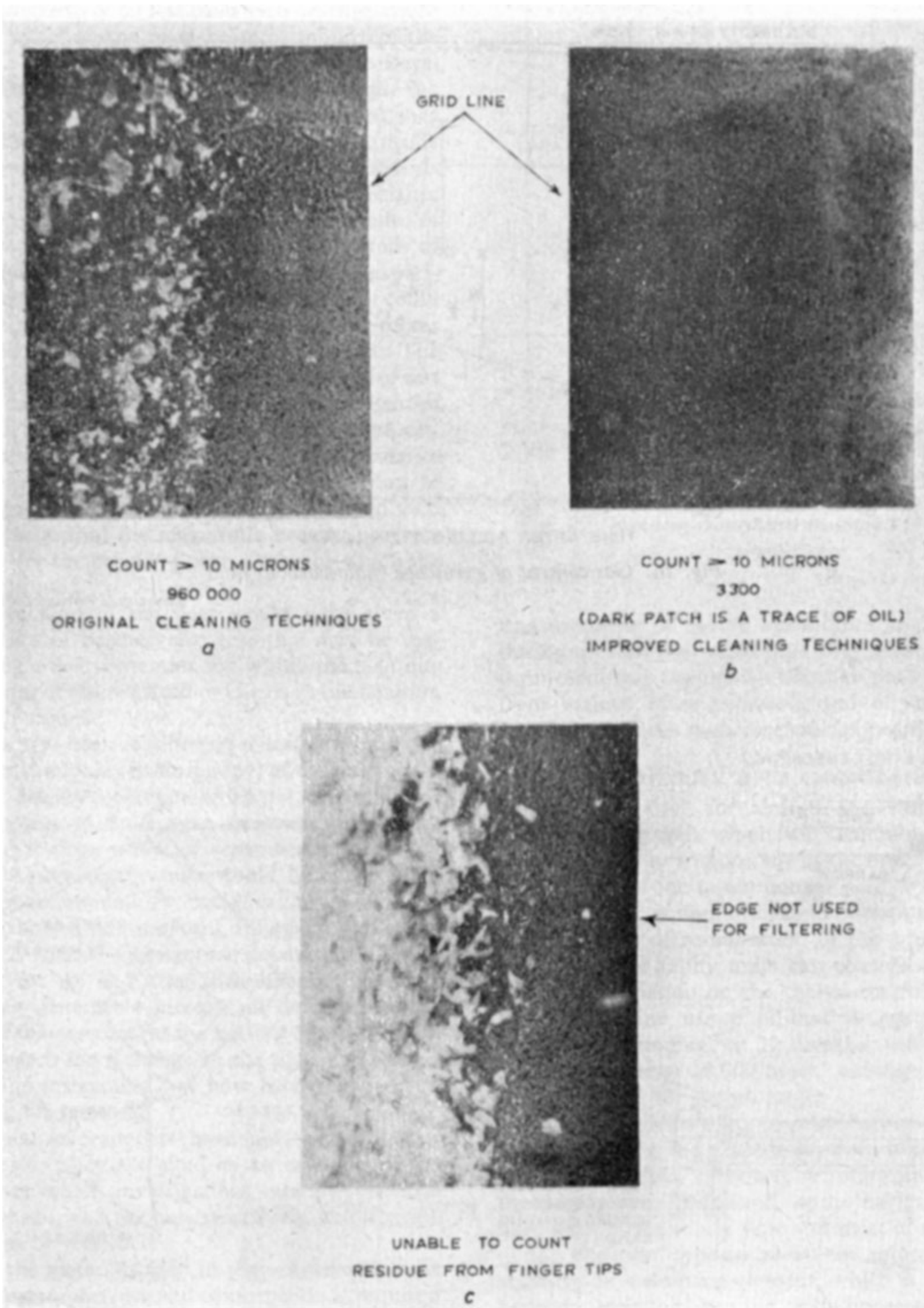


Fig. 25. Contamination of gyroscope fluid

(Courtesy of Ferranti Ltd)

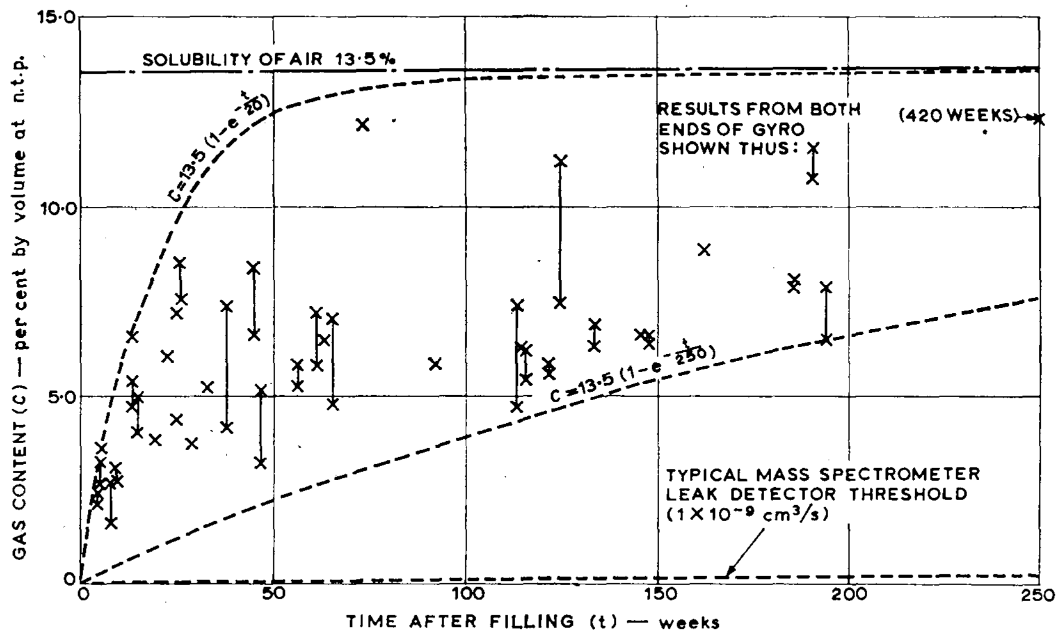
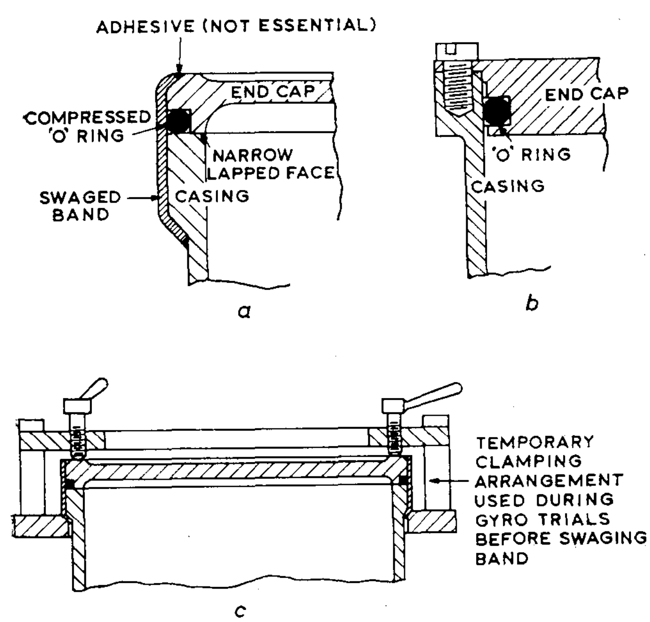


Fig. 26. Gas content of gyroscope (halo-carbon) fluid

(Courtesy of Ferranti Ltd)



a Two-stage seal.
b Standard piston ring seal.
c Two-stage seal; temporary clamp fitted.

Fig. 27. Casing seals

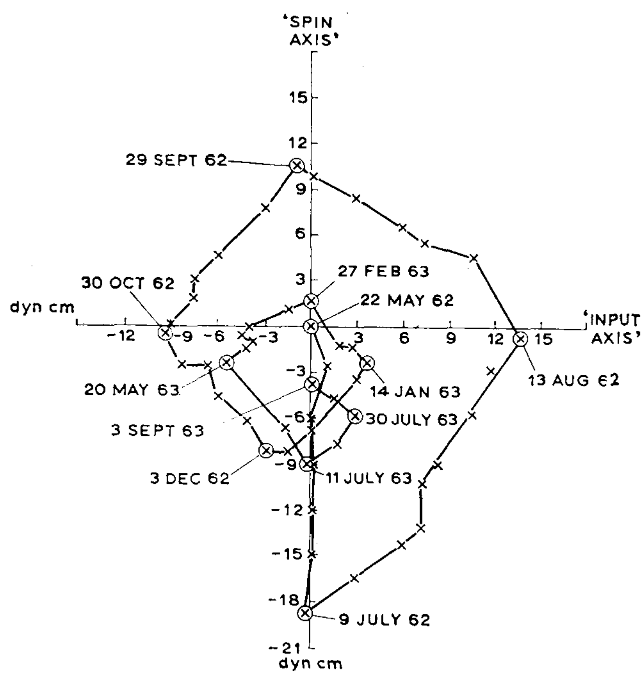


Fig. 28. Vector diagram of mass unbalance in instrument with dummy float

Systematic instabilities in performance

A comparison of Table 4 (column 8) with Fig. 16 shows that the few milligrams of oil lost from gyro bearings could have a serious effect on the mass-balance stability of the float structure. Tests have been in progress at the Royal Aircraft Establishment for a long period to investigate the possibility of oil migration being a source of balance instability. In the first experiments* the effect of attitude of rest on gyroscope mass unbalance was measured. It was found that the changes in the vector mass unbalance responded to the attitudes of rest, but the magnitudes of the changes were not commensurate with the periods of rest. It was then attempted to confirm by direct experiment that changes in balance of the sensitive element could be reduced, or even eliminated, by removing all the motor parts and lubricating oil from within the element. The results (Fig. 28) were surprising; in the first attitude of rest with the 'spin axis' vertical the unbalance torque changed progressively over a period of seven weeks by 19 dyn cm. The attitude of rest was changed by 90° and the balance change progressively shifted to follow the direction of gravity; this change with direction of rest continued with subsequent rotation, but the magnitude became progressively smaller over the period of over a year for which the experiment continued.

Plastic deformation of the float would have given a vectorial addition of balance changes; this may be discounted, leaving either contamination within the float due to a small amount of viscous fluid or effects in the flotation fluid as possible causes.

Experiments have been conducted† to test an hypothesis that thermo-gravitational separation (27) of the fluid could be the cause (a density separation of 0·6 per cent would be sufficient). Samples of fluid were extracted from those places in the gyroscope where, if separation were taking place, light or heavy components would be expected to collect. Density determinations, based on refractive index measurement, showed no significant differences (less than 0·1 per cent). A thermo-gravitational separating column has also been set up and after three months' running showed a barely detectable increase in density of fluid (of the order of 0·1 per cent) at the bottom of the column.

So far the search for a change in the fluid has proved negative and the instrument has now been reassembled with fresh fluid for retesting.

Although these experiments have not yet reached a definite conclusion they are cited as an example of the long period over which investigations into performance stability can extend, and the persistence required if cause and effect are to be related.

Stability of the materials used in the sensitive element is of fundamental importance and confirmation is required on the metallurgical stability of structural materials on which random movements may occur over long periods, as equilibrium is attained in the completion of phase changes, or as internal stresses are relieved. Fig. 29 shows measurements of severe instabilities in beryllium parts during the first few weeks following heat treatment. Work is proceeding to explore the effects of producing specimens from various billet positions, and of varying the heat treatment and the metallurgical composition.

* *By N. Sharp, J. Coldwell, and D. A. Carroll.*

† *By S. Scregg and A. T. Hill, Royal Aircraft Establishment.*

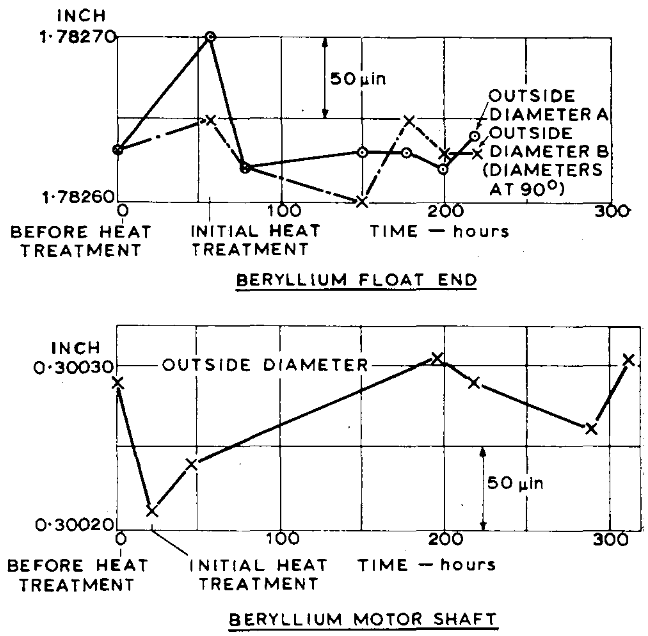


Fig. 29. Random movements measured in two beryllium components

(Results by courtesy of Ferranti Ltd)

SUMMARY AND CONCLUSIONS

Inertial navigation for aircraft and marine application requires gyroscopes which will maintain consistently an inertial datum to an accuracy of the order one arc minute for a period of one to ten hours.

Where there is high utilization of the navigation equipment the cost of recalibration of the gyroscope and the replacement of faulty units can considerably exceed the annual depreciation on the capital cost of the navigator. For civil airline use a calibration interval of at least 3500 hours' running, or 12 months, with a replacement interval of at least 10 000 hours' running, or three years, is suggested as the design target.

There is a wide variety of possible ways of producing a gyroscope using the Coriolis force on a 'measuring mass' which is in linear, vibratory, or rotary motion. So far all the instruments produced, with 'navigation' accuracy, have been of the rotary type and most of these have been of the 'flotation' type in which the rotor is encased in a sensitive or measuring element, which is supported by a hydrodynamic (air) bearing or by hydrostatic pressure.

Development potential of the flotation gyroscope

Examples have been given of the work in progress in the U.K. on obtaining a better understanding of the many factors that affect the life and stability of performance of the (hydrostatic) flotation gyroscope. The time cycle of such investigations is usually long, and, combined with the length of the manufacturing cycle, makes positive proof of the individual contribution from any investigation a lengthy task and difficult to obtain.

As an indication of what has already been achieved, however, results can be cited on one particular U.S. design manufactured in the U.K. The original rejection rate of U.S.-manufactured gyros when tested against the user's specification (more stringent than the manufacturer's) was 33 per cent; this rejection rate has been reduced in the U.K.-manufactured gyro to 5 per cent. The number of gyroscopes completed at the first assembly and fluid filling (i.e. no rework) has been increased from 44 per cent for the first U.K. production batches to 69 per cent (averaged over a sample of 170 units).

On spin-axis bearing life the investigations on ball bearings show promise of an overhaul life in excess of 10 000 hours being achieved; the choice between ball and gas spin axis bearings is likely for some time to remain an individual design choice determined by the particular application.

Improvements in stability of performance, and hence longer intervals between calibration obtains from better cleaning procedures, cleaner internal design, lower gas leakage, better material stability and other design factors which it has not been possible to cover in this paper. When tracked to source, elimination of systematic instabilities arising as a function of the rest position of the gyroscope should make further significant improvements to stability.

The performance aims cited appear to be within reach, by a continuation of systematic engineering investigation on the flotation type of instrument.

The 'next generation' gyroscope

In endeavouring to select the most profitable alternative lines for future developments, a clear distinction has to be made between achieving the ultimate in performance, with low cost, bulk, and weight as secondary factors, and achieving a more modest performance for applications where low cost, bulk, and weight are major factors.

For many applications in aircraft control and short-range missile guidance (3) the need exists for a cheap instrument of performance intermediate between that obtainable from the simple gyroscope (at a cost of a few hundred pounds) and the flotation instrument of navigation quality (at a cost of several thousand pounds). This need may be met by one of the more unconventional gyroscopes of Table 3; alternatively, increasing overhaul life and calibration interval of the flotation gyroscope may so lower the utilization cost that the flotation instrument becomes competitive with the 'unconventional' gyroscopes for applications where capital cost is not the dominant factor.

For applications where performance is the major consideration the potential advantages of the free-rotor instrument with a thin spherical shell are evident from Table 4. The indirect or flotation method of suspension, however, has the advantage that by containing the driving motor for the rotor within the measuring assembly, no torques result from the drive, provided that the wheel speed is constant. With direct suspension it is very difficult to avoid a component of the driving torque about the measuring axis, which tends to align the rotor to the driving axis with a short time constant, and experience so far is that the ultimate performance will only be obtained by allowing the rotor to coast freely in a vacuum.

Comparing the suspension problems, hydrostatic suspension starts with the immediate advantage of providing a force normal to the surface without any auxiliary equipment. To achieve this by electrostatic or superconducting-magnetic means involves considerable auxiliary apparatus. Although, therefore, a free-rotor gyroscope supported by one of the latter means may replace the flotation instrument where ultimate performance, rather than compactness, is the main objective, considerable development will be required to make such designs competitive in weight and volume with the small flotation instrument (4 and 5, Table 4).

Of all the possible new methods the 'laser' gyroscope is the most attractive; the problems of obtaining a performance comparable with 'rotor' gyroscopes, particularly within a comparable weight and volume, have as yet hardly been appreciated.

Whatever new method may be developed, however, the physical nature of the problem is such that extreme engineering precision and painstaking care and attention to minute details of design and assembly are likely to be needed.

ACKNOWLEDGEMENTS

The author is indebted to all those who have supplied material for incorporation in this lecture. Many interesting items have had to be excluded by pressure of space; it is emphasized that the items presented in Part III have been selected to illustrate particular problems and are not a complete account of flotation gyroscope research and development in the United Kingdom.

Particular thanks are due to Mr J. Drury and staff of Ferranti Ltd, Mr R. Hurrell and staff of British Aircraft Corporation Ltd, and Mr N. Sharp (now at Ministry of Aviation) and his former staff at the Royal Aircraft Establishment.

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